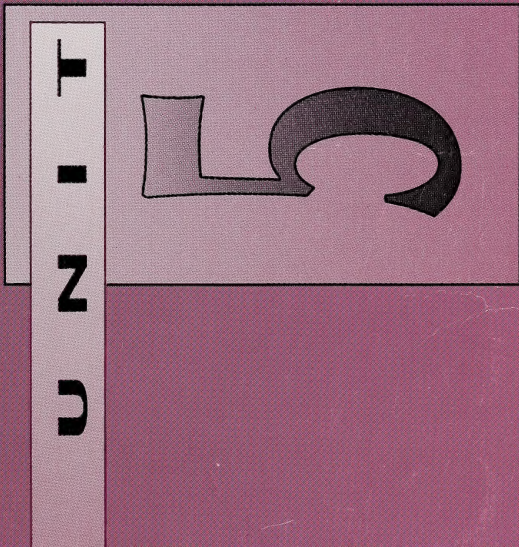




MATHEMATICS

10



COORDINATE GEOMETRY



Distance
Learning

Alberta
EDUCATION

W e l c o m e



Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

ALL RIGHTS RESERVED

Mathematics 10 Student Module Unit 5 Coordinate Geometry Alberta Distance Learning Centre ISBN No. 0-7741-0723-5

Copyright © 1992, the Crown in Right of Alberta, as represented by the Minister of Education, Alberta Education, 11160 Jasper Avenue, Edmonton, Alberta, T5K 0L2. All rights reserved. Additional copies may be obtained from the Learning Resources Distributing Centre.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Education.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this has not been done, please notify Alberta Education so appropriate corrective action can be taken.

This document is intended for	
Students	✓
Teachers (Mathematics 10)	✓
Administrators	
Parents	
General Public	
Other	

Cover photo: WESTFILE INC.

General Information

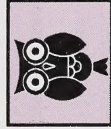
This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics (**Appendix B**, etc.).

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.



What You Already Know

- reviewing what you already know



Key Idea

- flagging important ideas



Audiotape

- learning by listening to an audiotape



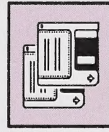
Review

- studying previous concepts



Another View

- exploring different perspectives



Computer Software

- learning by using computer software



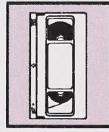
Introduction

- introducing the unit



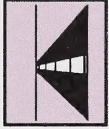
Solutions

- correcting the activities



Videotape

- learning by viewing a videotape



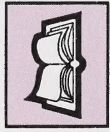
What Lies Ahead

- previewing the unit



Extra Help

- providing additional study



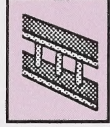
Print Pathway

- choosing a print alternative



Exploring the Topic

- actively learning new concepts



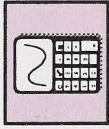
Extensions

- going on with the topic



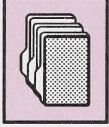
Calculator

- using your calculator



Graphing Calculator

- using your graphing calculator



What You Have Learned

- summarizing what you have learned

Mathematics 10

Course Overview

Mathematics 10 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Operations on Polynomials	14%
Unit 3 Equations and Inequalities	10%
Unit 4 Factoring Polynomials	13%
Unit 5 Coordinate Geometry	19%
Unit 6 Systems of Linear Equations	10%
Unit 7 Trigonometry	11%
Unit 8 Statistics	13%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Coordinate Geometry

This unit covers topics dealing with coordinate geometry. Each topic contains explanations, examples, and activities to assist you in understanding coordinate geometry. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 5 Coordinate Geometry

Contents at a Glance

Value	Coordinate Geometry	3
	What You Already Know	5
	Review	10
10%	Topic 1: Distance Between Two Points • Introduction • What Lies Ahead • Exploring Topic 1	11
10%	Topic 2: Finding the Midpoint • Introduction • What Lies Ahead • Exploring Topic 2	28
27%	Topic 3: Slope of a Line • Introduction • What Lies Ahead • Exploring Topic 3	38
12%	Topic 4: Graphing Linear Equations • Introduction • What Lies Ahead • Exploring Topic 4	71
25%	Topic 5: Slope and y-intercept • Introduction • What Lies Ahead • Exploring Topic 5	95
16%	Topic 6: Equations with Unique Conditions • Introduction • What Lies Ahead • Exploring Topic 6	119
	Unit Summary • What You Have Learned • Unit Assignment	139
	Appendices • Appendix A • Appendix B	140

Coordinate Geometry

You have studied some of the concepts of algebra in previous units. Although you have not yet studied geometry in any great detail, you will have some understanding from previous courses.

This unit will bridge the gap between geometry (lines) and algebra (numbers, equations, and formulas). For example, how would you find the slope of a ski run?

René Descartes invented the coordinate system to give a graphic picture of a number relation. This led to a dramatic change in mathematics and physics.

Unit 5

Coordinate Geometry

Topic 1
Distance Between
Two Points

Topic 2
Finding the
Midpoint

Topic 3
Slope of a Line

Topic 4
Graphing Linear
Equations

Topic 5
Slope and
 y -intercept

Topic 6
Equations
with
Unique
Conditions

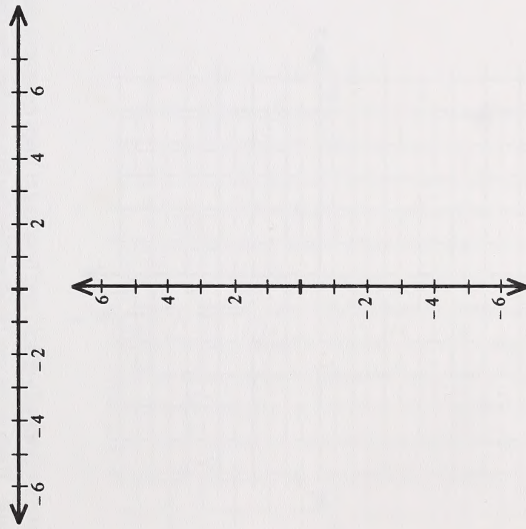


What You Already Know

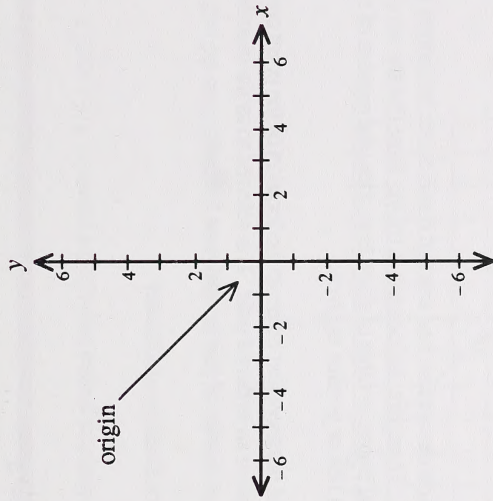
Recall the following:

- Terminology used in graphing on the Cartesian Plane.

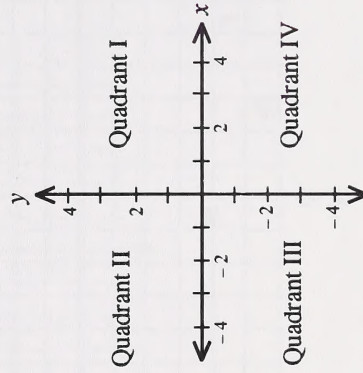
If you place a horizontal number line over a vertical number line so that they intersect at zero, you create a coordinate plane. Each line is called an **axis**. (Axes is plural for axis.)



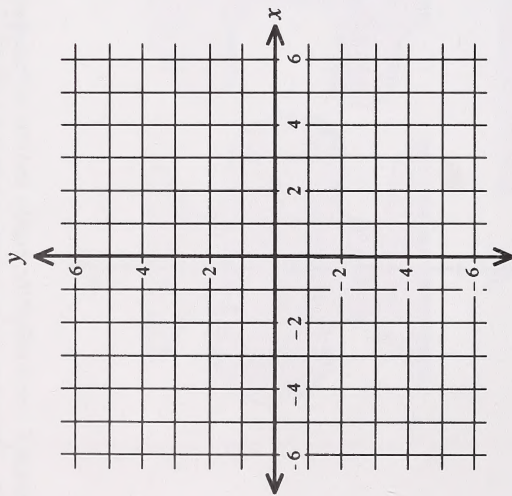
The horizontal line is called the **x-axis**, and the vertical line is called the **y-axis**. The point of intersection is called the **origin**.



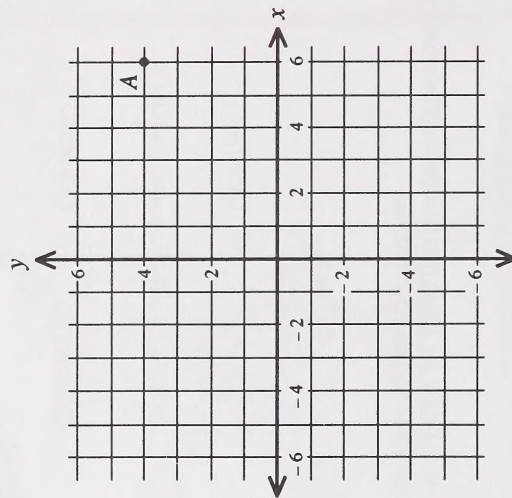
These two axes divide the plane into four parts called **quadrants**.
The four quadrants are named as follows:



Vertical and horizontal lines drawn through each point on the axes are included to make it easier to locate points on the plane.



The intersections of vertical and horizontal lines represent points on the plane. Each point can be identified by the number the vertical line it is on and the number the horizontal line it is on. For example, look at the point marked *A* on the following plane.



The number of the vertical line is called the *x*-value, and the number of the horizontal line is called the *y*-value.

Point *A* is on the vertical line marked 6 and on the horizontal line marked 4. Therefore, its location is (6, 4). Note that the number of the vertical line or *x*-value is given first; then, the number of the horizontal line or *y*-value is given.

The pair of numbers that refers to the location of the point on the plane is called an **ordered pair**. It is referred to as the **coordinates** of the point. An ordered pair is written as follows:

(*x*-coordinate, *y*-coordinate)

The x -coordinate is sometimes called the **abscissa**. The y -coordinate is sometimes called the **ordinate**.

(abscissa, ordinate)

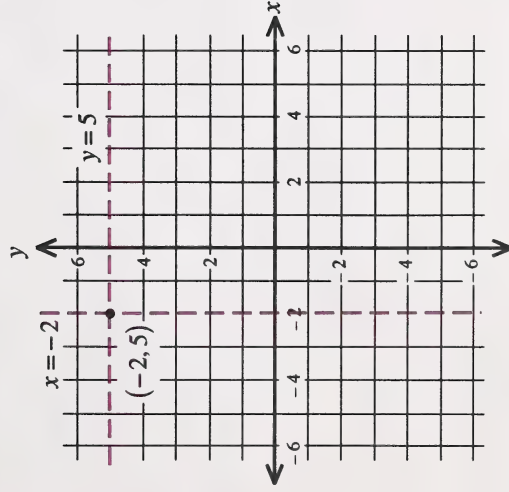
The coordinates of the origin are $(0, 0)$.

- Graphing points and lines.

Example 1

Plot the point $(-2, 5)$.

Solution:

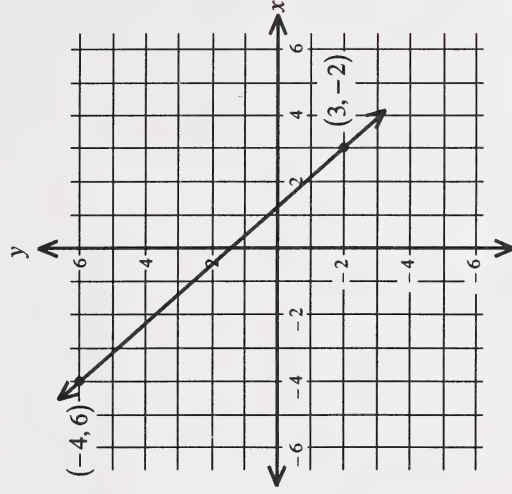


Example 2

Graph the line that passes through the points $(-4, 6)$ and $(3, -2)$.

Solution:

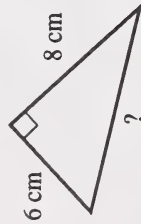
Plot the two given points, and draw a straight line through both points.



- Pythagorean theorem

Example 3

Find the length of the missing side of the following triangle.



Solution:

$$c^2 = a^2 + b^2$$

$$c^2 = (6)^2 + (8)^2$$

$$c^2 = 36 + 64$$

$$c^2 = 100$$

$$c = \sqrt{100}$$

$$= 10 \text{ cm}$$

The length of the missing side is 10 cm.

- Plotting points that satisfy linear equations.

Example 4

Plot the points that satisfy the equation $y = 2x - 4$.

Solution:

Make a table of values by choosing a number to replace x ; then calculate the corresponding value for y .

$$\begin{aligned} \text{When } x = 0, y &= 2(0) - 4 \\ &= 0 - 4 \\ &= -4 \end{aligned}$$

$$\begin{aligned} \text{When } x = -2, y &= 2(-2) - 4 \\ &= -4 - 4 \\ &= -8 \end{aligned}$$

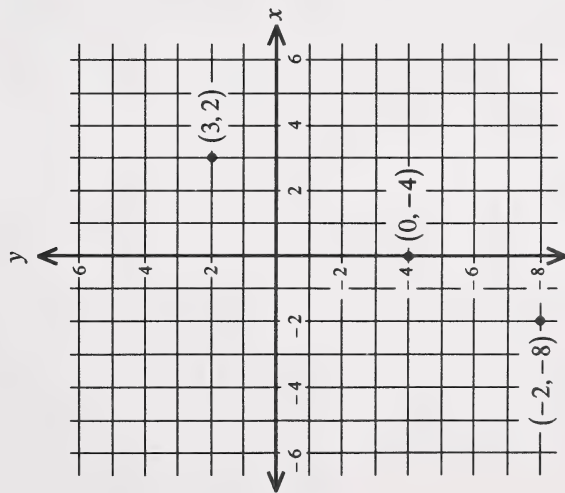
$$\begin{aligned} \text{When } x = 3, y &= 2(3) - 4 \\ &= 6 - 4 \\ &= 2 \end{aligned}$$

Table of values

x	0	-2	3
y	-4	-8	2

Plot each of the ordered pairs.

$(0, -4), (-2, -8), (3, 2)$



- Change subjects in formulas.

Example 5

Rewrite the formula $A = \frac{1}{2}a(b_1 + b_2)$ for b_2 .

Solution:

$$A = \frac{1}{2}a(b_1 + b_2)$$

$$2(A) = 2 \times \frac{1}{2}a(b_1 + b_2) \quad (\text{Multiply both sides by 2.})$$

$$2A = a(b_1 + b_2)$$

$$\frac{2A}{a} = \frac{a(b_1 + b_2)}{a} \quad (\text{Divide both sides by } a.)$$

$$\frac{2A}{a} = b_1 + b_2$$

$$\frac{2A}{a} - b_1 = b_1 + b_2 - b_1 \quad (\text{Subtract } b_1 \text{ from both sides.})$$

$$\frac{2A}{a} - b_1 = b_2, \text{ or}$$

$$b_2 = \frac{2A}{a} - b_1$$

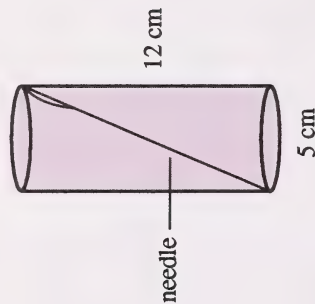
Now that you have reviewed some material you studied previously, do the following **Review**.



Review

Complete the following questions. Graph paper is provided in **Appendix B**.

1. Find the length of the longest needle that can be placed in a cylinder that has a diameter of 5 cm and a height of 12 cm.



2. Plot the points $A(0, 4)$, $B(4, -5)$, $C(-2, -4)$, and $D(-4, 4)$ on the grid.
3. Solve for y when $y = 3x + 4$ and $x = -3$.



Now go to the **Review** solutions in **Appendix A**.

If you had difficulties with the review questions, go to **Mathematics 9** and review the appropriate sections.

Topic 1 Distance Between Two Points

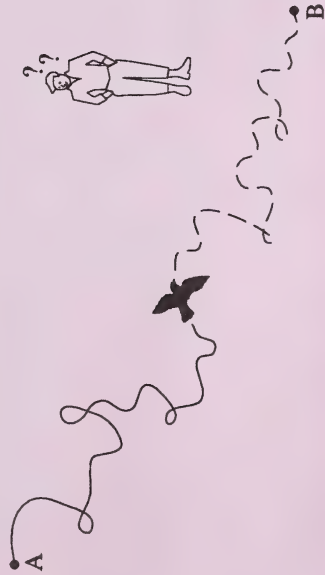


Introduction

Maps in an atlas and most maps in a geography text have grids. Why do astronomers use grids to identify positions of stars in the sky? By using grids you can find the distance between two points. Cities, streets, and buildings are located using the coordinate system. The Cartesian coordinate system locates points in the plane.

Read on and you will discover a way to find the distance between any two points in a plane.

What is the distance from A to B as the crow flies?



What Lies Ahead

Throughout the topic you will learn to

1. find the distance between two points
2. develop and use the distance formula to solve problems

Now that you know what to expect, turn the page to begin your study of distance between two points.



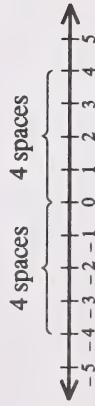
Exploring Topic 1

Activity 1



Find the distance between two points.

Before finding the distance between two points, you must understand absolute value. Look at the following number line.



Look at the distance between -4 and 4 . You can say that -4 is located four spaces from 0 , and that 4 is located four spaces from 0 . You can also write this in absolute value form:

$|-4| = 4$ and $|4| = 4$. Now you can say the following:



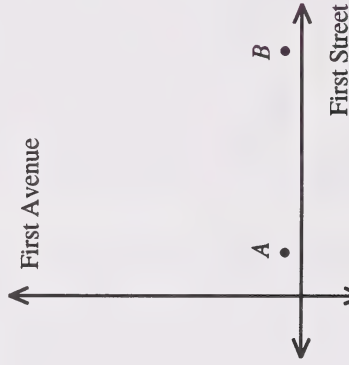
The absolute value of a number is its distance from 0 on a number line.

Therefore, the distance from -4 to 4 is eight spaces.

The absolute value is a distance, not direction; thus, a sign is not needed. Now see how the distance between two points can be found by using absolute value.

Example 1

The two axes in the diagram represent a street and an avenue in a town. A fire station is located 30 m from First avenue at A , and a police station is located at B . If the police station is 320 m from First avenue and $\overline{AB}$ is parallel to First street, how far is B from A ?



Note: The absolute value of a number does not use a sign. The absolute value of a **positive** or **negative** number is **positive**. The absolute value of **zero** is **zero**.

The two bars mean the **absolute value** of. $|-4|$ means the **absolute value of -4** . $|4|$ means the **absolute of 4** .

$$|-4| = 4 \text{ and } |4| = 4$$

Solution:

The distance from A to B is the difference between the distance of B from First Avenue and the distance of A from First Avenue.

Distance is defined as the absolute value of the difference between two numbers. The order of subtraction does not matter.

$$\begin{aligned} AB &= |320 - 30| \quad \text{or} \quad AB = |30 - 320| \\ &= |290| \\ &= 290 \end{aligned}$$

B is 290 m from A .

Notice that sometimes there is a bar over the letters naming the endpoints of a line segment and sometimes there is not.

Whenever working with line segments, use a bar over the letters. For example, find the length of $\overline{PQ}$, or the length of $\overline{PQ}$ is 8 units. But when specifically talking about the measure or distance, no bar is used. For example,

$$\begin{aligned} PQ &= 8 \text{ units or } AB = |320 - 20|. \\ &= |290| \\ &= 290 \end{aligned}$$

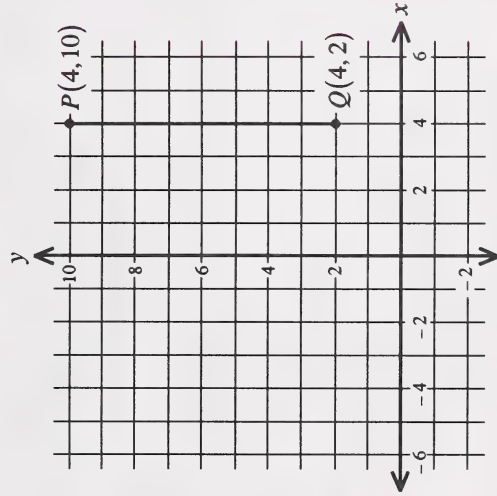
Now look at the next two examples to see how you can use the coordinate system to find the length of a line segment.

Example 2

$P(4, 10)$ and $Q(4, 2)$ are two points in the coordinate system. Find the length of $\overline{PQ}$.

Solution:

By inspection, and with the help of the y -axis, the length of $\overline{PQ}$ is 8 units. You can also calculate the length of $\overline{PQ}$. P is 10 units from the x -axis and Q is 2 units from the x -axis. The difference is 8 units $(10 - 2)$. On close examination you will notice the difference between the **unlike values** of the two coordinates, $|10 - 2| = 8$.



$AB = |\text{distance from } B \text{ to First avenue} - \text{distance from } A \text{ to First avenue}|$

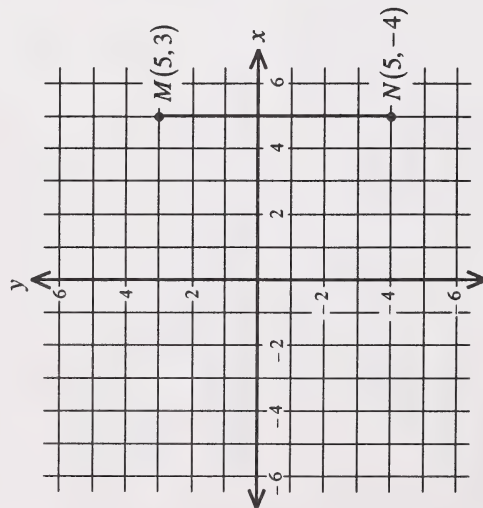
In the ordered pairs $(4, 10)$ and $(4, 2)$, 10 and 2 are the **unlike values**.

Length is always a positive number.

Look at the following example to find the length of a line segment whose endpoints are in different quadrants.

Example 3

In the following diagram, $\overline{MN}$ is a line segment parallel to the y -axis. Calculate the length of $\overline{MN}$.



Solution:

Find the length of $\overline{MN}$ by examining the location of points M and N . M is 3 units from the x -axis. N is 4 units from the x -axis. The sum of 3 and 4 is 7.

The same result is obtained by finding the difference between the unlike values of the coordinates of M and N . Now, $-4 - 3 = -7$. Length is always a positive value; therefore, you must find the absolute value of the difference.

$$|-4 - 3| = |-7| \\ = 7$$

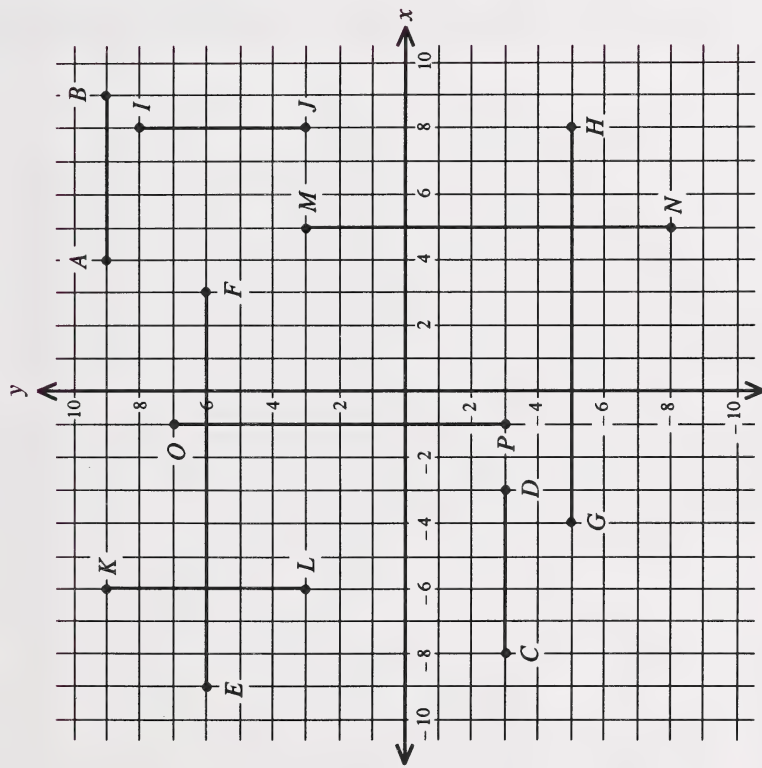
From this point on, you will follow this rule to find the length or distance between two points.

1. Find the length of the following segments from the diagram which follows. Do either the left or the right column.

- | | |
|--------------------|--------------------|
| a. $\overline{AB}$ | b. $\overline{CD}$ |
| c. $\overline{EF}$ | d. $\overline{GH}$ |
| e. $\overline{IJ}$ | f. $\overline{KL}$ |
| g. $\overline{MN}$ | h. $\overline{OP}$ |

For absolute value, the order of subtraction does not matter because you get the same result.

$$|3 - (-4)| = |3 + 4| \\ = 7$$



For solutions to Activity 1, turn to Appendix A, Topic 1.

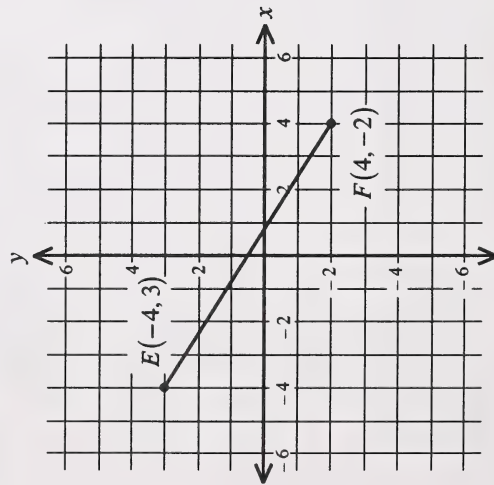
In the previous examples you found the lengths of line segments parallel to the x -axis or the y -axis.

What happens when line segments are not parallel?

Once again the Pythagorean theorem is handy. The next example shows you how to find the length of a line segment that is not parallel to the x -axis or the y -axis.

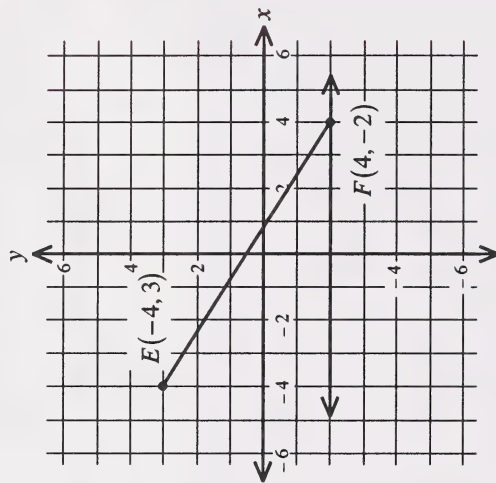
Example 4

Find the distance between the points $E(-4, 3)$ and $F(4, -2)$.



Solution:

Draw a line parallel to the x -axis passing through point F .

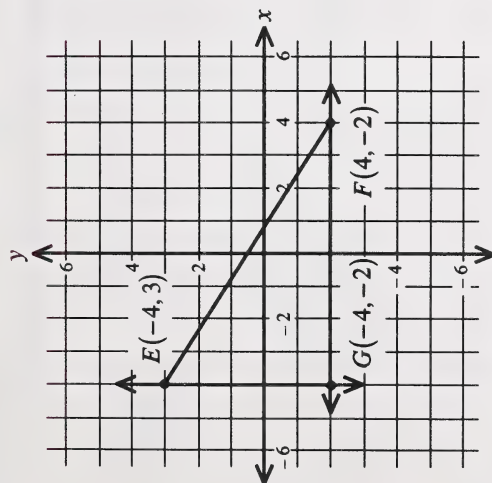


Draw another line parallel to the y -axis passing through point E . Call the point of intersection of these two lines point G whose coordinates are $(-4, -2)$.

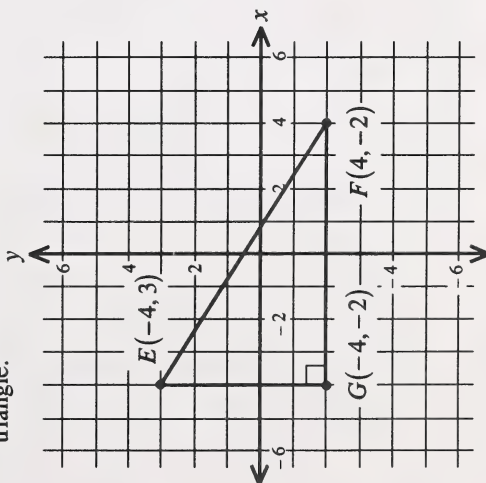
Since G is on the same vertical line as E , its first coordinate is the same as that of point E . That is, -4 .

Since G is on the same horizontal line as F , its second coordinate is the same as that of point F . That is, -2 .

Therefore, the coordinates of G are $(-4, -2)$.



The resulting triangle ($\triangle EFG$) is a right triangle.



Using the rule you learned earlier for segments parallel to the x -axis or the y -axis, calculate the lengths of $\overline{EG}$ and $\overline{GF}$.

$$\begin{aligned} EG &= |3 - (-2)| \\ &= 5 \end{aligned}$$

$$\begin{aligned} GF &= |-4 - 4| \\ &= 8 \end{aligned}$$



$\overline{EG}$ is parallel to the y -axis; therefore, the y -coordinates are used to find its length. Similarly, $\overline{GF}$ is parallel to the x -axis; therefore, the x -coordinates are used to find its length.

Now you can use the Pythagorean theorem.

$$\begin{aligned}(EF)^2 &= (EG)^2 + (GF)^2 \\(EF)^2 &= |-2-3|^2 + |4-(-4)|^2 \\(EF)^2 &= |-5|^2 + |8|^2 \\(EF)^2 &= 25 + 64 \\(EF)^2 &= 89 \\\sqrt{(EF)^2} &= \sqrt{89} \\EF &\doteq 9.4\end{aligned}$$

or

$$\begin{aligned}g^2 &= 5^2 + 8^2 \\g^2 &= 25 + 64 \\g^2 &= 89 \\\sqrt{g^2} &= \sqrt{89} \\g &\doteq 9.4\end{aligned}$$

The length of $\overline{EF}$ is approximately 9.4 units.

Try the following questions.

2. Do a and c or b and d. You may plot the points on grid paper if you find it helpful. Graph paper is provided in **Appendix B**.
 - a. $A(-2, 7)$ and $B(8, 7)$ are two points in a coordinate system. Find the length of $\overline{AB}$.
 - b. The points $P(4, -8)$ and $Q(4, 16)$ lie on a line. Find the length of $\overline{PQ}$.
 - c. Find the distance between the points $E(3, -5)$ and $F(-4, 8)$. Round your answer to the nearest tenth.
 - d. $A(4, 6)$ and $B(-2, -6)$ are two points on a line. Find the distance between A and B . Round your answer to the nearest tenth.



For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

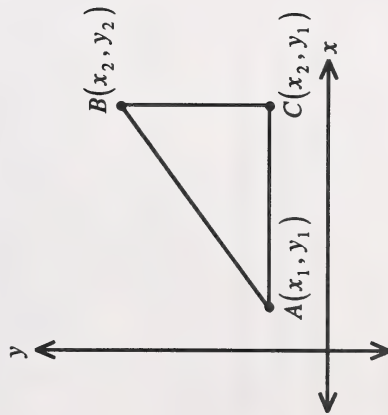
Activity 2



Develop and use the distance formula to solve problems.

In Example 4, you learned a method to find the length of an oblique line segment. Using this idea, you will develop a formula to find the length of a line segment that is not parallel to the x -axis or the y -axis.

The following diagram shows the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in a plane. ABC is a right triangle with a right angle at point C . The coordinates of C are (x_2, y_1) .



The length of $\overline{AC}$ is $|x_2 - x_1|$.

The length of $\overline{BC}$ is $|y_2 - y_1|$.

Using the Pythagorean theorem,

$$\begin{aligned}(AB)^2 &= (AC)^2 + (BC)^2 \\ &= |x_2 - x_1|^2 + |y_2 - y_1|^2\end{aligned}$$

Now find the square root on both sides of the equation to get AB .

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are any two points in a plane, the distance between A and B is given by the following formula:



$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula can be used, not only for oblique line segments, but for line segments that are parallel to the x - or y -axis.

x_2 and x_1 could be interchanged and this would not affect the formula. For example, $|x_2 - x_1| = |x_1 - x_2|$.

An oblique line is a line that is not parallel to either the vertical axis or the horizontal axis.

$$\begin{aligned}|x_2 - x_1|^2 &= (x_2 - x_1)^2 \\ |y_2 - y_1|^2 &= (y_2 - y_1)^2\end{aligned}$$

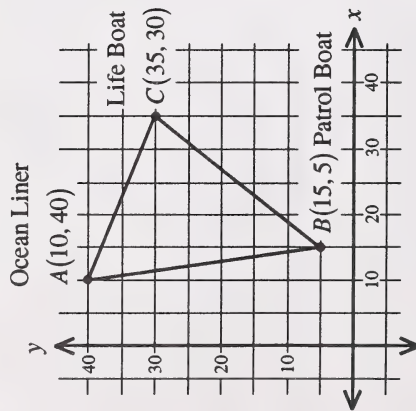
$$\begin{aligned}(x_2 - x_1)^2 &= (x_1 - x_2)^2 \\ (y_2 - y_1)^2 &= (y_1 - y_2)^2\end{aligned}$$

$\overline{AC}$ is parallel to the x -axis; thus, the x -coordinates are used to find its distance. $\overline{BC}$ is parallel to the y -axis; thus, the y -coordinates are used to find its length.

The next example shows you an application of this formula.

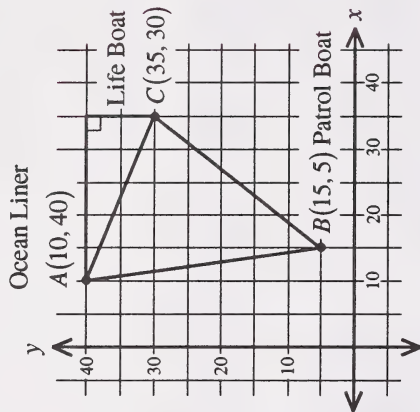
Example 5

A patrol boat located at $B(15 \text{ km}, 5 \text{ km})$ receives a message from an ocean liner at $A(10 \text{ km}, 40 \text{ km})$. A life boat is spotted at $C(35 \text{ km}, 30 \text{ km})$.



- Which vessel is closer to the life boat?

Solution:



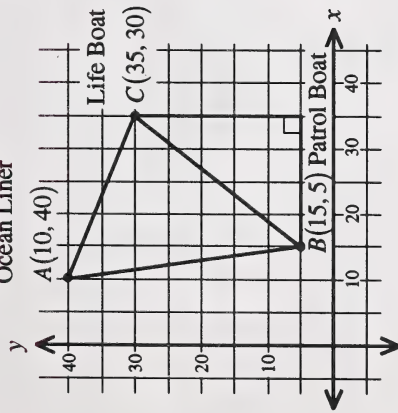
Triangle ABC may not be a right triangle.

$$\begin{aligned}
 AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(35 - 10)^2 + (30 - 40)^2} \\
 &= \sqrt{(25)^2 + (-10)^2} \\
 &= \sqrt{625 + 100} \\
 &= \sqrt{725} \\
 &\approx 27
 \end{aligned}$$

The distance between the ocean liner and the life boat is about 27 km.

Use the distance formula to find the distance of $\overline{AC}$, $\overline{BC}$, and $\overline{AB}$. The formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Ocean Liner



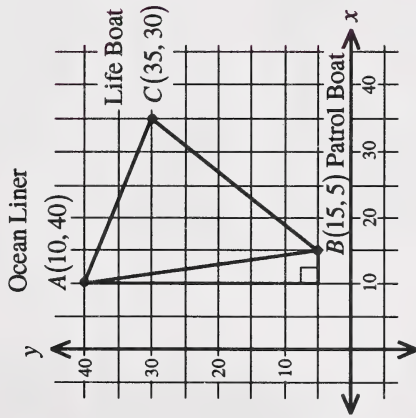
$$\begin{aligned}
 BC &= \sqrt{(35-15)^2 + (30-5)^2} \\
 &= \sqrt{(20)^2 + (25)^2} \\
 &= \sqrt{400 + 625} \\
 &= \sqrt{1025} \\
 &\doteq 32
 \end{aligned}$$

The distance between the patrol boat and the life boat is about 32 km.

Therefore, the ocean liner is closer to the life boat.

- How far apart are the ocean liner and the patrol boat?

Solution:



$$\begin{aligned}
 AB &= \sqrt{(15-10)^2 + (5-40)^2} \\
 &= \sqrt{(5)^2 + (-35)^2} \\
 &= \sqrt{25 + 1225} \\
 &= \sqrt{1250} \\
 &\doteq 35
 \end{aligned}$$

The ocean liner and the patrol boat are approximately 35 km apart.

If $A(x_1, y_1)$ and $B(x_2, y_2)$,
then

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

where AB represents the
distance between point A and
point B .

Now try the following questions. Be sure to review if you are having difficulty. Do questions 1c, 2, and 3. If you need more practice, complete 1a and 1b. Graph paper is provided in **Appendix B**.

1. Calculate the distance between each pair of points. Round your answer to one decimal place.

a. $(5, -2)$ and $(0, 6)$

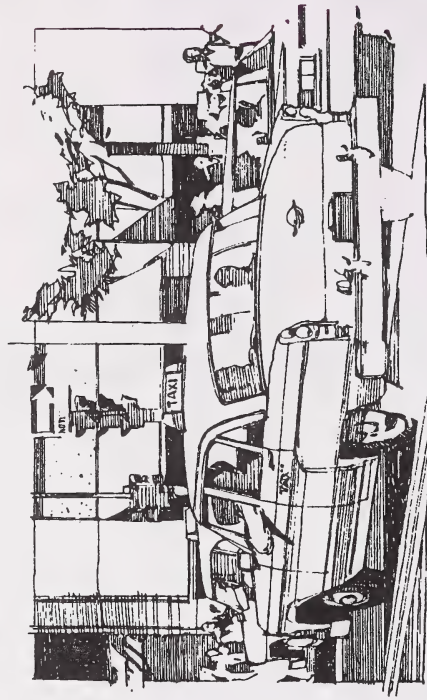
b. $(-4, 7)$ and $(-3, -8)$

c. $(3, 0)$ and $(1, -2.5)$

2. On a grid, draw the triangle with vertices $D(-3, 8)$, $F(-3, -4)$, and $E(3, 2)$. Find the length of each side and show that it is an isosceles triangle.



3. Two highways in a town, 1 and 2, intersect at right angles. Let highway 1 be the x -axis and highway 2 the y -axis. A professional building is located 2.5 km from highway 1 and 1 km from highway 2. A hospital is at $(3 \text{ km}, 0.5 \text{ km})$. What is the distance between the two buildings to the nearest whole number? The only access to both buildings is directly from the highways. How many kilometres will you cover if you go from one building to the other?



For solutions to Activity 2, turn to Appendix A,
Topic 1.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

1. Answer the following questions by filling in the word or phrase.

a. The two bars on either side of -7 in $|-7|$ means _____.

b. The absolute value of a positive or negative number is _____.

c. Length is always a _____.

d. If $d^2 = e^2 + f^2$ represents the Pythagorean theorem, then _____ is the right angle in $\triangle DEF$.

2. Do either a or b.

a. $A(-2, 5)$ and $B(-3, -3)$ are two points on a line. Find the distance between A and B using the distance formula. Round your answer to the nearest tenth.

b. Using the Pythagorean theorem, find the length of $\overline{DF}$ where D has coordinates $(-4, 2)$ and F has coordinates $(6, 8)$. Round your answer to the nearest tenth.

3. Plot the following points on a grid. Connect the points in each group (a, b, c, d, e) in the order given. Graph paper is provided in Appendix B.

- | | | |
|----|--------------------|-------------------|
| a. | i. $(9, 7)$ | ii. $(13, 11)$ |
| | iii. $(12, 12)$ | iv. $(2, 5)$ |
| | v. $(-2, -1)$ | vi. $(0, -3)$ |
| | vii. $(8, 0)$ | viii. $(10, 3)$ |
| | ix. $(11, 6.5)$ | |
| b. | i. $(5, -6)$ | ii. $(4, -5)$ |
| | iii. $(1, -6)$ | iv. $(-8, -6)$ |
| | v. $(-9, -8)$ | vi. $(-9, -6)$ |
| | vii. $(-11, -7)$ | viii. $(-10, -5)$ |
| | ix. $(-11, -4)$ | x. $(-11, -2)$ |
| | xi. $(-9, 0)$ | xii. $(-5, 2)$ |
| | xiii. $(5, 11)$ | xiv. $(9, 24)$ |
| | xv. $(14, 20)$ | xvi. $(8, 11)$ |
| | xvii. $(8.5, 9.6)$ | |

- c. i. $(1, -6)$ ii. $(0.5, -8)$
 iii. $(-2, -8)$ iv. $(0.5, -8)$
 v. $(-1.5, -8.5)$ vi. $(0.5, -8)$
 vii. $(-1, -9)$
- d. i. $\left(6\frac{6}{7}, -3\frac{3}{7}\right)$ ii. $(4, -10)$
 iii. $\left(4\frac{2}{3}, -8\right)$ iv. $(2.5, -8.5)$
 v. $\left(4\frac{2}{3}, -8\right)$ vi. $(3, -9.5)$
 vii. $\left(4\frac{2}{3}, -8\right)$
- e. i. $(8.5, 4)$ ii. $(11.5, 7)$
 iii. $(12, 10)$

To complete the figure, plot a circle with the center at $(-8.5, -3.5)$.
 The radius of the circle is 0.5 units. Colour the interior of the circle.



For solutions to **Extra Help**, turn to **Appendix A**,
Topic 1.



Extensions

When using the distance formula to find the distance between two points on a coordinate plane, the coordinates for x and y do not have to be whole numbers. Study the examples that follow to see how approximate distances can be calculated when the coordinates are fractional numbers, decimal numerals, or radicals. Extensive use of a calculator is recommended when working with these types of values.

Example 6

Find the distances between points A and B when the coordinates for these points are as follows. Approximate to the nearest hundredth using a calculator.

• $A = \left(2\frac{3}{4}, -4\frac{1}{2}\right)$ and $B = \left(-3\frac{1}{3}, 5\frac{2}{5}\right)$

Solution:

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(-3\frac{1}{3} - 2\frac{3}{4}\right)^2 + \left[5\frac{2}{5} - \left(-4\frac{1}{2}\right)\right]^2} \\
 &= \sqrt{\left(-3\frac{4}{12} - 2\frac{9}{12}\right)^2 + \left(5\frac{2}{5} + 4\frac{1}{2}\right)^2} \\
 &= \sqrt{\left(-5\frac{13}{12}\right)^2 + \left(5\frac{4}{10} + 4\frac{5}{10}\right)^2} \\
 &= \sqrt{\left(-6\frac{1}{12}\right)^2 + \left(9\frac{9}{10}\right)^2} \\
 &= \sqrt{\left(\frac{-73}{12}\right)^2 + \left(\frac{99}{10}\right)^2} \\
 &= \sqrt{(-6.08\bar{3})^2 + (9.9)^2} \\
 &= \sqrt{37.006\,944\,44 + 98.01} \\
 &= \sqrt{135.016\,944\,44} \\
 &\doteq 11.619\,679\,19 \\
 &\doteq 11.62
 \end{aligned}$$

The distance between points A and B is approximately 11.62 units or $11\frac{31}{50}$ units.

- $A = (4.36, 6.04)$ and $B = (-2.52, -6.44)$

Solution:

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-2.52 - 4.36)^2 + (-6.44 - 6.04)^2} \\
 &= \sqrt{(-6.88)^2 + (-12.48)^2} \\
 &= \sqrt{47.3344 + 155.7504} \\
 &= \sqrt{203.0848} \\
 &\doteq 14.250\,782\,43 \\
 &\doteq 14.25
 \end{aligned}$$

The distance between points A and B is approximately 14.25 units.

It is time to try some similar problems on your own. Do any two of the following.

1. In each of the following find the distance between points A and B using the distance formula.

a. $A = \left(-4\frac{1}{3}, -6\frac{1}{2}\right)$ $B = \left(5\frac{1}{2}, -1\frac{3}{4}\right)$

b. $A = \left(6\frac{1}{5}, 3\frac{1}{4}\right)$ $B = \left(-1\frac{1}{8}, -6\frac{5}{9}\right)$

c. $A = (-4.3, -6.5)$ $B = (5.5, -1.75)$

d. $A = (6.2, 3.25)$ $B = (-1.125, -6.56)$

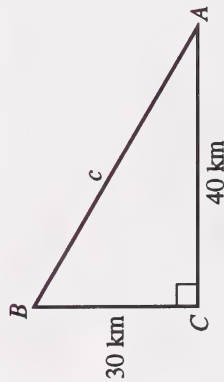


For solutions to **Extensions**, turn to **Appendix A**,
Topic 1.

Sometimes you may be asked to determine how long it takes to go a certain distance. If you are given the rate of speed (r) and the distance travelled (d), you can calculate the time required (t) using the formula $t = \frac{d}{r}$. Study the next example; then answer the question that follows using your knowledge of the Pythagorean theorem.

Example 7

How long does it take to drive from A to B assuming that the rate of speed is 80 km/h ?



Solution:

Use the Pythagorean theorem to find the distance from A to B .

$$c^2 = a^2 + b^2$$

$$c^2 = 30^2 + 40^2$$

$$c^2 = 900 + 1600$$

$$c^2 = 2500$$

$$c = \sqrt{2500}$$

$$= 50 \text{ km}$$

Then use the formula $t = \frac{d}{r}$.

$$t = \frac{d}{r}$$

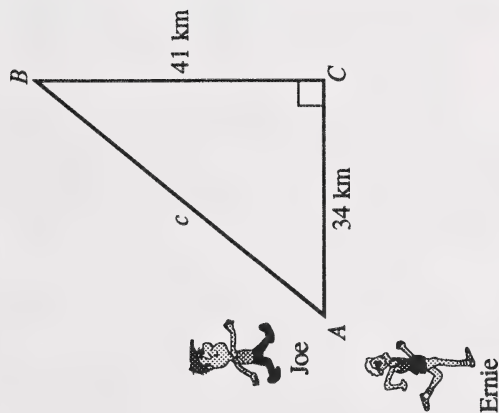
$$= \frac{50 \text{ km}}{80 \text{ km/h}}$$

$$= \frac{5}{8} \text{ h or } 0.625 \text{ h}$$

To change 0.625 h to minutes, multiply 0.625 by 60 to get $0.625 \times 60 = 37.5 \text{ min}$. It would take $\frac{5}{8} \text{ h}$ or 37.5 min to drive from A to B .

Try the following question.

2. Joe is going to walk directly from Point A to Point B at the rate of 5 km/h. Ernie is going to run from Point A to Point C and then to Point B at the rate of 7.5 km/h. If they leave at the same time, who will get to Point B first?



For solutions to Extensions, turn to Appendix A, Topic 1.

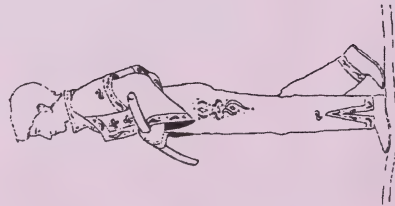
Topic 2 Finding the Midpoint



Introduction

To balance a uniform beam, at which point is it pivoted? Where does a high-wire performer grip the balance pole for stability? In both cases the answer is at the midpoint.

Explore the relationship between the coordinates of the midpoint of a line segment and the coordinates of the endpoints.



What Lies Ahead

Throughout the topic you will learn to

1. develop and use the midpoint formula for finding the midpoint of a line segment
2. solve problems that involve the use of the midpoint formula

Now that you know what to expect, turn the page to begin your study of finding the midpoint.



Exploring Topic 2

Activity 1



Develop and use the midpoint formula for finding the midpoint of a line segment.

Look at the following line. It has two numerical values.



You can find the midpoint between these two numbers either by inspection or by calculation. Calculation is faster.

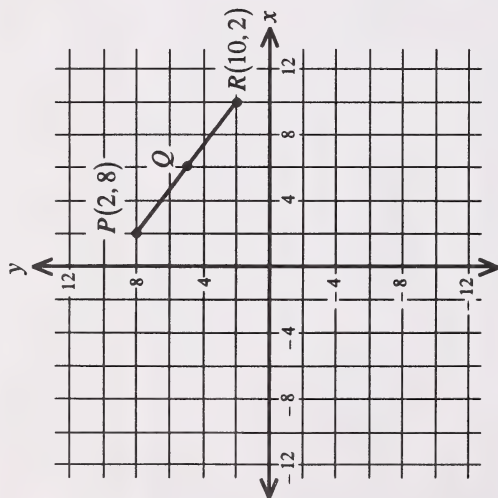
The midpoint is obtained by adding 2 to 16, then dividing by 2. The number midway between 2 and 16 is $\frac{2+16}{2} = 9$.

The midpoint is the average of the two endpoints. You will extend this idea to finding the midpoint of a line segment in the Cartesian plane.



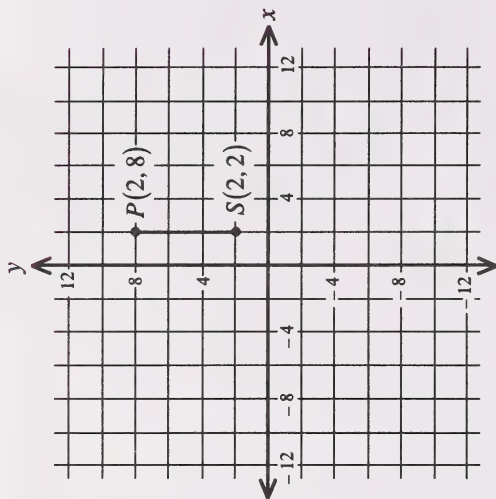
Example 1

Q is the midpoint of a line segment joining $P(2, 8)$ and $R(10, 2)$. Find the coordinates of Q .



Solution:

Find the midpoint of $\overline{PS}$.

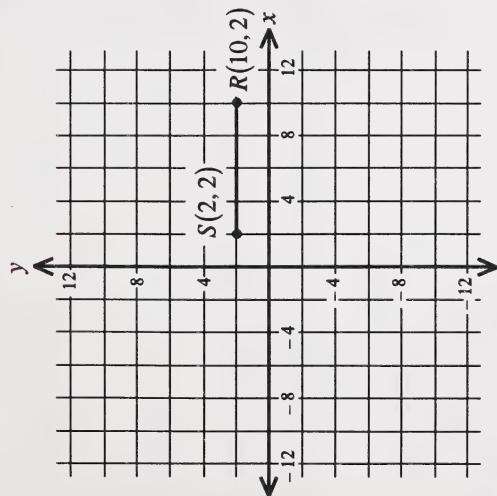


Since P and S have the same first coordinate, find the average of the second (y -coordinate) coordinates.

$$\frac{8+2}{2} = 5$$

The midpoint of $\overline{PS}$ is $(2, 5)$.

Find the midpoint of $\overline{SR}$.



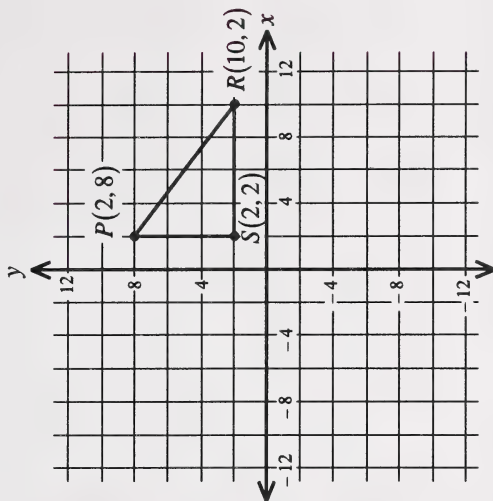
Since S and R have the same second coordinate, find the average of the first (x -coordinate) coordinates.

$$\frac{2+10}{2} = 6$$

The midpoint of $\overline{SR}$ is $(6, 2)$.

Now calculate the coordinates of Q .

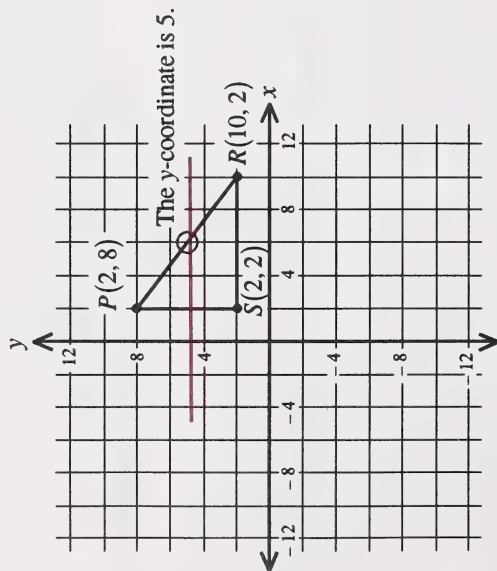
Complete the triangle which includes $\overline{PR}$.



$\triangle PSR$ is a right triangle. The coordinates of S are $(2, 2)$.

The y-coordinate of the midpoint of $\overline{PS}$ is 5.

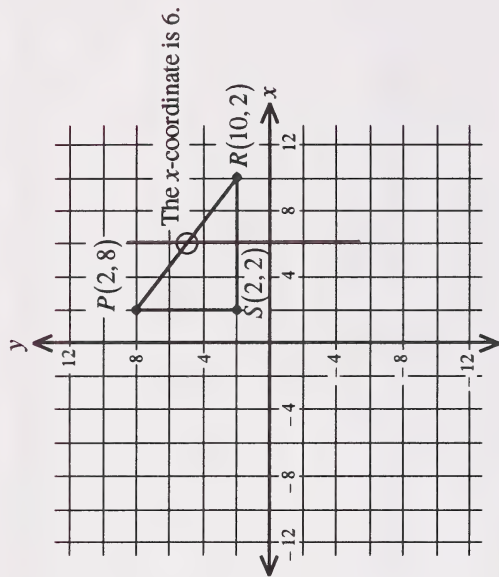
Draw the line $y = 5$. This line passes through the midpoint of segment $\overline{PR}$.



Therefore, the y-coordinate of the midpoint of $\overline{PR}$ is 5.

The x-coordinate of the midpoint of $\overline{SR}$ is 6.

Draw the line $x = 6$. This line also passes through the midpoint of segment $\overline{PR}$.



Therefore, the x-coordinate of the midpoint of $\overline{PR}$ is 6.

The coordinates of Q are $(6, 5)$.

You can now obtain a general formula to find the coordinates of the midpoint from the coordinates of the endpoints.

If P represents the point whose coordinates are (x_1, y_1) and R represents the point whose coordinates are (x_2, y_2) , then the coordinates of the midpoint of $\overline{PR}$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



Look at the next example to see how this formula is used.

Example 2

If $A(-8, -6)$ and $B(7, -10)$ are the endpoints of a line segment, find the coordinates of the midpoint of $\overline{AB}$.

Solution:

$$\begin{aligned} \text{x-coordinate} &= \frac{x_1 + x_2}{2} \\ &= \frac{-8 + 7}{2} \\ &= \frac{-1}{2} \\ &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{y-coordinate} &= \frac{y_1 + y_2}{2} \\ &= \frac{-6 + (-10)}{2} \\ &= \frac{-16}{2} \\ &= -8 \end{aligned}$$

The midpoint of $\overline{AB}$ is $(-\frac{1}{2}, -8)$.

Do the following questions.

Do at least 1, 2b, 2c, and 3.

1. $A(12, -2)$, $B(6, 4)$, and $C(0, 10)$ are three points. Show that B is the midpoint of $\overline{AC}$.
2. Determine the coordinates of the midpoint of $\overline{EG}$ for the following.
 - a. $E(7, 5)$ and $G(11, -2)$
 - b. $E(6, 4)$ and $G(-10, 14)$
 - c. $E(-5, -9)$ and $G(-8, -6)$
 - d. $E(83, -16)$ and $G(45, 20)$

3. Show that $(2, -7)$ is equidistant from

$$M(-5, -9) \text{ and } N(9, -5).$$



For solutions to **Activity 1**, turn to **Appendix A, Topic 2**.

If C is the midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$C = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{or } C = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right).$$

Equidistant means the same distance apart.

Activity 2



Solve problems that involve the use of the midpoint formula.

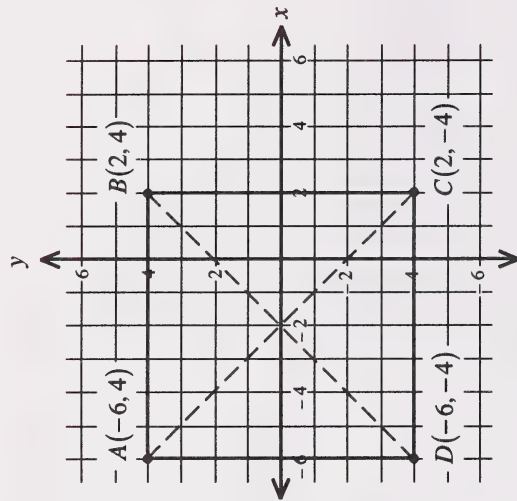
Study the following example to see how the midpoint formula is used to solve problems.

Example 3

$A(-6, 4)$, $B(2, 4)$, $C(2, -4)$, and $D(-6, -4)$ are the coordinates of the vertices of a square. Find the coordinates of the midpoints of its diagonals. How do they compare?

Solution:

In a word problem of this type you should always draw a diagram.



The diagonals are $\overline{AC}$ and $\overline{BD}$.

The midpoint of $\overline{AC}$ is found by using the following formula.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{-6 + 2}{2}, \frac{4 + (-4)}{2} \right) \\ &= \left(\frac{-4}{2}, \frac{0}{2} \right) \\ &= (-2, 0) \end{aligned}$$

The midpoint of $\overline{BD}$ is as follows:

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}, \frac{y_2 + y_1}{2}\right) &= \left(\frac{2 + (-6)}{2}, \frac{4 + (-4)}{2}\right) \\ &= \left(\frac{-4}{2}, \frac{0}{2}\right) \\ &= (-2, 0)\end{aligned}$$

The diagonals have a common midpoint.

Do any three questions. Draw a diagram first. Graph paper is provided in **Appendix B**.

1. Triangle ABC has vertices $A(-3, 5)$, $B(2, 6)$, and $C(-6, -2)$. Calculate the distance from A to the midpoint of the opposite side to one decimal place.
2. The endpoints of the diameter of a circle has coordinates $(-2, 4)$ and $(6, -8)$. Find the centre of the circle and the length of its radius to the nearest whole number.
3. A parallelogram has vertices $M(0, -3)$, $A(7, -1)$, $T(10, 4)$, and $H(3, 2)$. Plot the parallelogram on a grid and find the coordinates of the midpoints of its two diagonals. What do you notice?
4. The coordinates of the vertices of $\triangle DEF$ are $D(6, 6)$, $E(-4, 2)$, and $F(8, -2)$. G is the midpoint of $\overline{DE}$, and H is the midpoint of $\overline{DF}$. What do you notice about the lengths of $\overline{GH}$ and $\overline{EF}$?



For solutions to **Activity 2**, turn to **Appendix A, Topic 2**.

The segment that joins a vertex and the midpoint of the opposite side of a triangle is called a median.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.



Extra Help

$\overline{AB}$ is part of a number line. C is midway between A and B .



The midpoint between the numbers -4 and 2 is -1 . This is the average of -4 and 2 .

$$\frac{-4 + 2}{2} = -1$$

$M(-4, 6)$ and $N(2, -8)$ are two points on a line in a coordinate plane. The midpoint of $\overline{MN}$ is the average of the x -coordinates and the average of the y -coordinates.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{-4 + 2}{2}, \frac{6 + (-8)}{2} \right) \\ &= \left(\frac{-2}{2}, \frac{-2}{2} \right) \\ &= (-1, -1) \end{aligned}$$

Do question 1; then do question 2 or 3. Graph paper is provided in **Appendix B**.

1. State whether each statement is true or false.
 - a. The midpoint between $(0, 0)$ and $(8, 2)$ is $(4, 2)$.
 - b. The number midway between -10 and 2 is -4 .
 - c. $(-1.7, 4)$ is equidistant from $(-8, 6)$ and $(4.6, 2.0)$.
 - d. The midpoint of the segment joining $(3, 7)$ and $(9, -3)$ is $(5, -3)$.
2. The coordinates of the vertices of a rectangle are $A(2, 2)$, $B(10, 2)$, $C(10, 8)$, and $D(2, 8)$. Draw the rectangle on a grid and find the coordinates of the midpoints of its two diagonals. What do you notice?
3. On a grid, draw the triangle with vertices $D(8, 4)$, $E(4, -2)$, and $F(-6, 10)$. Find the coordinates of the midpoint of each side.



For solutions to **Extra Help**, turn to **Appendix A**, **Topic 2**.



Extensions

If you know the midpoint and an endpoint of a line segment, you can calculate the other endpoint of this line segment.

Example 4

For example, the line segment PQ has one endpoint $P(-4, 8)$ and a midpoint $(6, 12)$. Find the coordinates of point Q .

Solution:

Use the midpoint formula, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Let $(x_1, y_1) = (-4, 8)$.

By substitution $\frac{-4+x_2}{2} = 6$ and $\frac{8+y_2}{2} = 12$.

Solve for x_2 and y_2 .

$$\begin{aligned} -4 + x_2 &= 12 & 8 + y_2 &= 24 \\ x_2 &= 12 + 4 & y_2 &= 24 - 8 \\ & & &= 16 \end{aligned}$$

The coordinates of Q are $(16, 16)$.

Do any two questions.

1. Find the coordinates of three points that divide $\overline{MN}$ into four equal parts when the coordinates of $\overline{MN}$ are $(-6, 2)$ and $(8, -12)$.
2. $M(-3, 6)$ is the midpoint of $\overline{AB}$. If the coordinates of endpoint A are $(8, 10)$, find the coordinates of the other endpoint B .
3. ABC is a triangle. Point A has coordinates $(2, 8)$. The coordinates of the midpoint of $\overline{AB}$ are $(4, 2)$, and the coordinates of the midpoint of $\overline{BC}$ are $(0, -4)$. Find the coordinates of B and C .



For solutions to **Extensions**, turn to **Appendix A, Topic 2**.



Topic 3 Slope of a Line



Introduction

The roof of a house must have a certain steepness so that the snow will slide off. How do you measure the steepness of the roof on your house? How are you able to show that one mountain is steeper than another?

How can you show that lines are perpendicular and how can you test for parallelism?

To answer these questions you will examine the concept of **slope**.

Work through this topic to understand the concept of slope.



What Lies Ahead

Throughout the topic you will learn to

1. define slope of a line segment as rise over run
2. develop and apply the formula for slope
3. identify and use the relationship between slopes of parallel lines
4. identify and use the relationship between slopes of perpendicular lines
5. identify collinear points and use slope to determine if points in a plane are collinear

Now that you know what to expect, turn the page to begin your study of slope of a line.



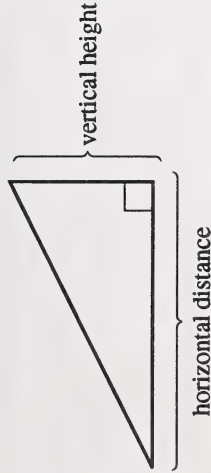
Exploring Topic 3

Activity 1



Define slope of a line segment as rise over run.

The slope or steepness of an inclined plane can be found by the ratio $\frac{\text{vertical height}}{\text{horizontal distance}}$.



Consider the following inclined plane.



The slope can be found by the ratio $\frac{5}{15} = \frac{1}{3}$.

The slope of the inclined plane is $\frac{1}{3}$.

Note that slope has no units; the units cancel out.

$$\text{slope} = \frac{1}{3} = \frac{\text{vertical height}}{\text{horizontal distance}}.$$

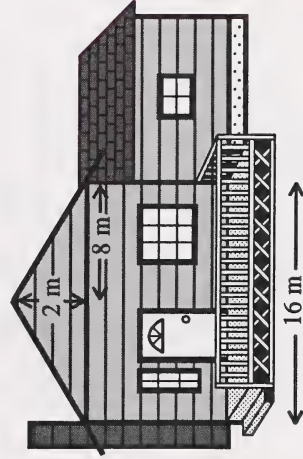
You can conclude that for every 1 m of vertical height there must be 3 m of horizontal distance.

Use the following relation to find slope.



$m = \frac{\text{rise}}{\text{run}}$, where m is the slope, rise is the vertical height, and run is the horizontal distance.

Use this relation to calculate the steepness (slope) of the roof of a house.



The run in this example is 8 m. That is half the width of the house which is 16 m.

You are given the vertical height and the horizontal distance of the roof.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2\text{ m}}{8\text{ m}} \\ &= \frac{1}{4} \end{aligned}$$

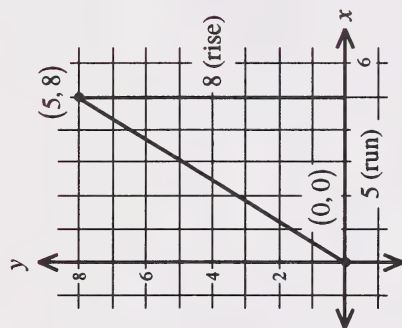
The slope is $\frac{1}{4}$.

You can see that for every 1 m of rise, there are 4 m of run.

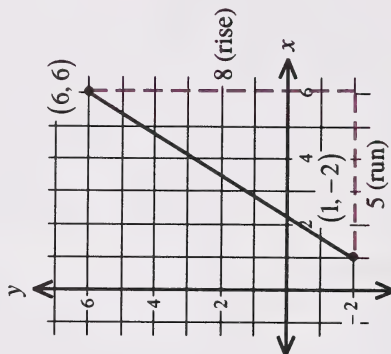
The concept of slope = $\frac{\text{rise}}{\text{run}}$ can be used to find the slope of a line segment.

Consider the line segment whose endpoints are $(0, 0)$ and $(5, 8)$. Find the slope of this line segment. Since $(0, 0)$ is at the origin, the endpoint $(5, 8)$ gives the vertical and horizontal distance. The vertical height is 8 and the horizontal distance is 5. Therefore, the rise is 8 units and the run is 5 units.

$$m = \frac{8}{5}$$



The same rise and run can be obtained from different coordinates.



Always draw a diagram if possible. It is much easier to visualize the situation with a diagram.

The distance between two points on a vertical line or a horizontal line is the absolute value of the difference of the unlike values of the coordinates of the two points.

Now go to a more difficult example.

Example 1

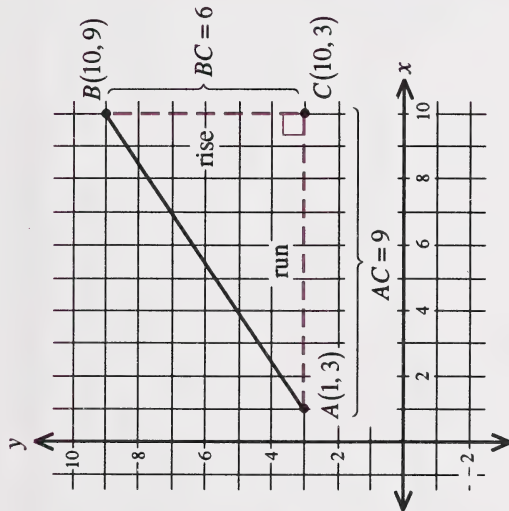
Find the slope of a line segment whose endpoints are $A(1, 3)$ and $B(10, 9)$.

Solution:

First draw a diagram.

Before you can find the slope, you have to find the coordinates of point C .

The coordinates of point C can be found by projecting point A horizontally to the right and projecting point B downward. The point of intersection for these two projections is $(10, 3)$. Therefore, point C has coordinates $(10, 3)$.



To calculate the slope of this segment, you need to calculate the $\frac{\text{rise}}{\text{run}}$.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{BC}{AC} \\ &= \frac{6}{9} \\ &= \frac{2}{3} \end{aligned}$$

The slope is $\frac{2}{3}$. This means that for every 2 units of vertical rise there must be 3 units of horizontal run.

$\overline{BC}$ is a vertical line segment. Therefore, BC (distance from B to C) can be found by the following:

$$\begin{aligned} |9 - 3| &= |6| \\ &= 6 \end{aligned}$$

$\overline{AC}$ is a horizontal line segment. Therefore, AC (distance from A to C) can be found by the following:

$$\begin{aligned} |1 - 10| &= |-9| \\ &= 9 \end{aligned}$$

$BC = 6$, which represents the rise. $AC = 9$, which represents the run.

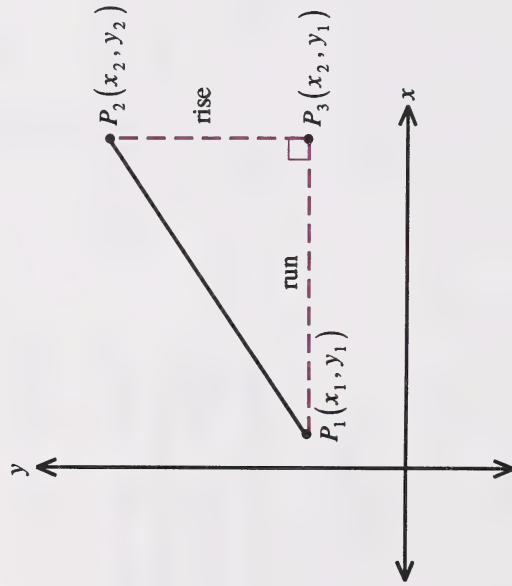
Activity 2



Develop and apply the formula for slope.

You now know how to find the slope of a line segment if the two endpoints are given. Now you will derive a general formula to find the slope of a line segment.

Find the slope of the line segment whose endpoints are at $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.



You will need the coordinates of P_3 to find the rise and run.

$$\text{Now } m = \frac{\text{rise}}{\text{run}} = \frac{P_2 P_3}{P_1 P_3}.$$

The first coordinate of P_3 has to be the same as the first coordinate of P_2 because P_2 and P_3 form a vertical segment.

You can also see that the second coordinate of P_3 has to be the same as the second coordinate of P_1 , because P_1 and P_3 form a horizontal segment.

$P_2 P_3$ = distance between P_2 and P_3

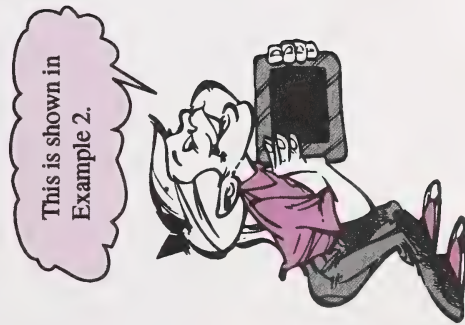
$P_1 P_3$ = distance between P_1 and P_3

The difference in the y -coordinates gives the rise $(y_2 - y_1)$. The difference in the x -coordinates gives the run $(x_2 - x_1)$. The vertical segment P_2P_3 is equal to $y_2 - y_1$. The horizontal segment P_1P_3 is equal to $x_2 - x_1$. Therefore, $m = \frac{y_2 - y_1}{x_2 - x_1}$, where m denotes slope.



The slope of the line segment joining two points is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$.

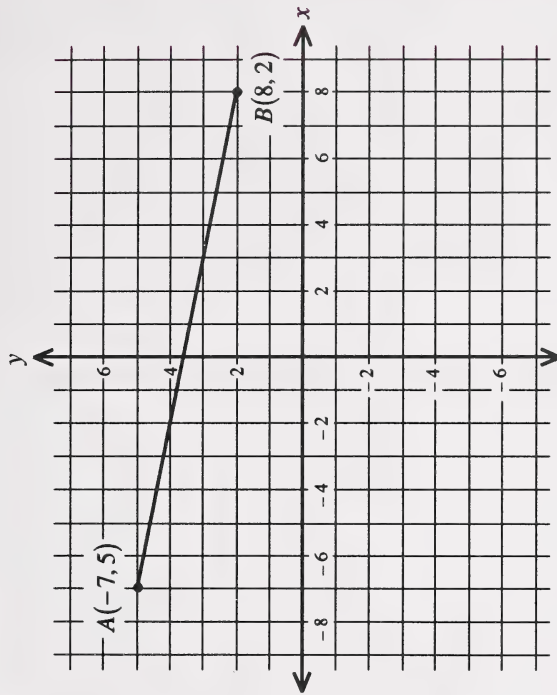
When you find the slope of a line segment, it does not matter which point is considered the first point and which is considered the second point.



Example 2

Find the slope of the line segment whose endpoints are $A(-7, 5)$ and $B(8, 2)$.

Solution:



Let $A(-7, 5)$ be the first point, and let $B(8, 2)$ be the second point.

$$\begin{aligned} m_{\overline{AB}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 5}{8 - (-7)} \\ &= \frac{-3}{15} \\ &= -\frac{1}{5} \end{aligned}$$

Now let $B(8, 2)$ be the first point, and let $A(-7, 5)$ be the second point.

$$\begin{aligned} m_{\overline{BA}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 2}{-7 - 8} \\ &= \frac{3}{-15} \\ &= \frac{1}{-5} \\ &= -\frac{1}{5} \end{aligned}$$

The slope of $\overline{AB}$ is $-\frac{1}{5}$.

$m_{\overline{AB}}$ means the slope of line segment $\overline{AB}$.

$$\overline{AB} = \overline{BA}$$

Now do the following questions.

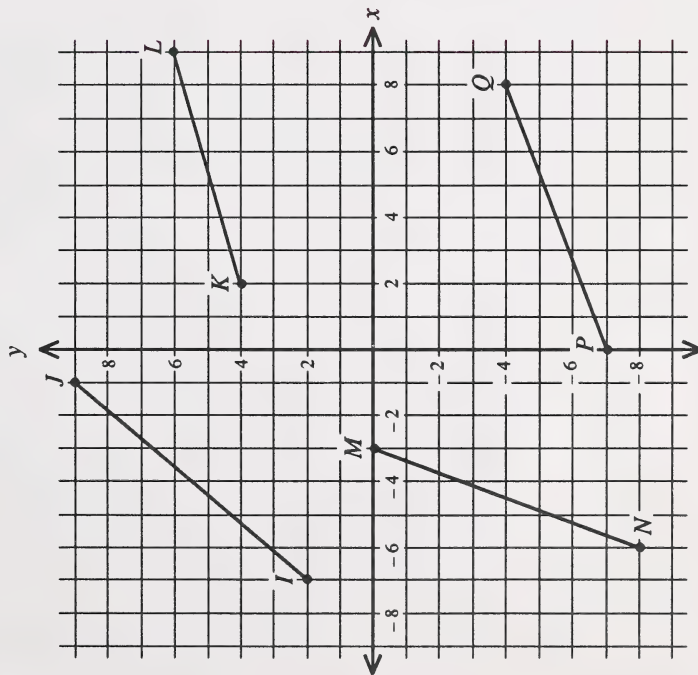
- Calculate the slopes of the following line segments. Do a and c or b and d.

a. $m_{\overline{IJ}}$

b. $m_{\overline{KL}}$

c. $m_{\overline{MN}}$

d. $m_{\overline{PQ}}$



$m_{\overline{IJ}}$ means the slope of line segment $\overline{IJ}$.

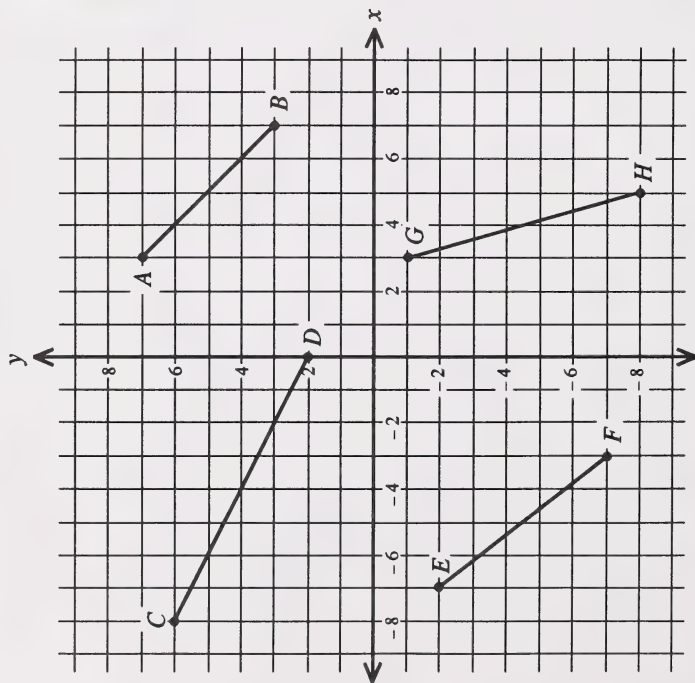
2. Calculate the slopes of the following line segments. Do a and c or b and d.

a. $m_{\overline{CD}}$

b. $m_{\overline{GH}}$

c. $m_{\overline{AB}}$

d. $m_{\overline{EF}}$



3. a. What do the slopes of the segments in question 1 have in common?
b. What do the segments of question 1 have in common?
4. a. What do the slopes of the segments in question 2 have in common?
b. What do the segments of question 2 have in common?

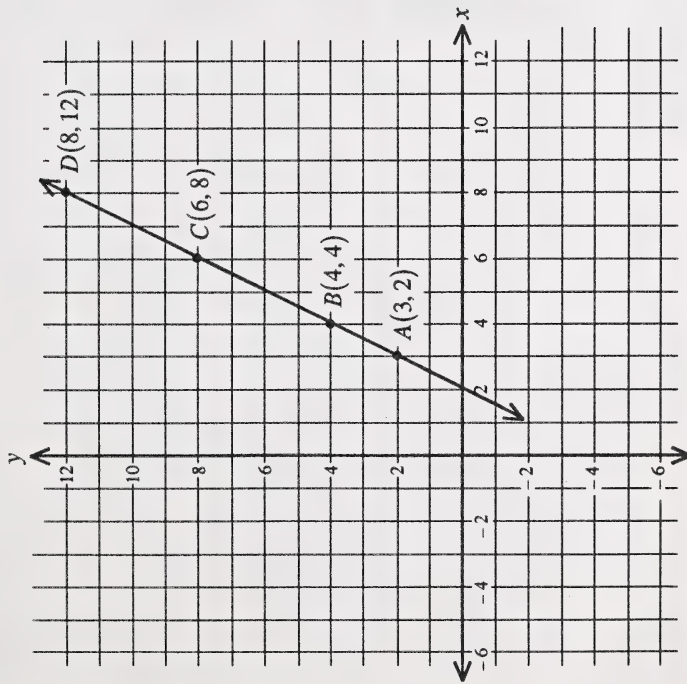


For solutions to **Activity 2**, turn to **Appendix A, Topic 3**.

You have just seen that the slope of a line segment can be determined by using the coordinates of the endpoints. You will extend the concept of slope of a segment to slope of a line.



The points A, B, C, and D lie on a line. You will find the slope of this line using the coordinates of any pair of points.



The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\begin{aligned}\text{The slope of } \overline{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 2}{4 - 3} \\ &= \frac{2}{1} \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{The slope of } \overline{AD} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{12 - 2}{8 - 3} \\ &= \frac{10}{5} \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{The slope of } \overline{BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - 4}{6 - 4} \\ &= \frac{4}{2} \\ &= 2\end{aligned}$$

Since the slopes are equal, 2 is the slope of every segment of this line. You can conclude that the slopes of all segments of a line are equal.

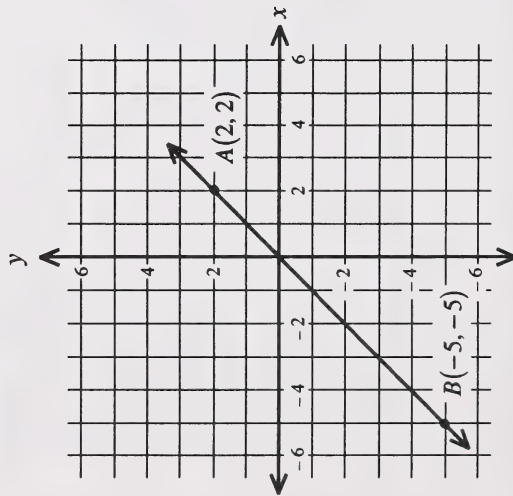
The sign of a slope determines the direction of a line. Look at the next example.

Example 3

Graph, determine the slope, and determine the direction of a line running through the following points.

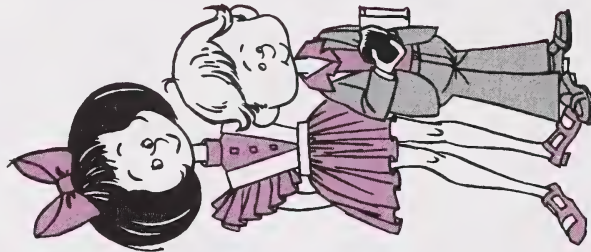
- $A(2, 2)$ and $B(-5, -5)$.

Solution:



$$\begin{aligned} m_{\overline{AB}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 2}{-5 - 2} \\ &= \frac{-7}{-7} \\ &= 1 \end{aligned}$$

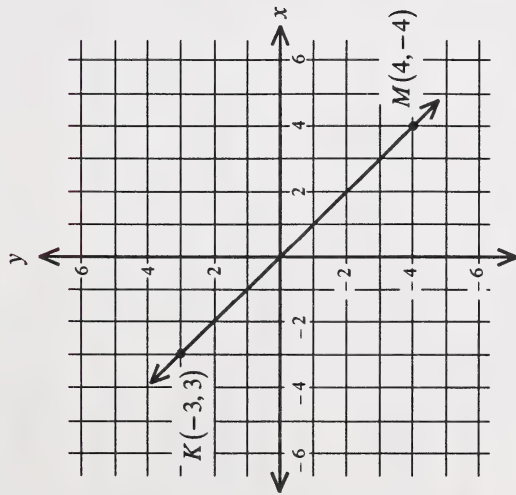
The slope is 1; therefore, the graph rises to the right.



A positive slope means that the line goes upward from left to right. A negative slope means that the line goes downward from left to right.

- $K(-3, 3)$ and $M(4, -4)$

Solution:

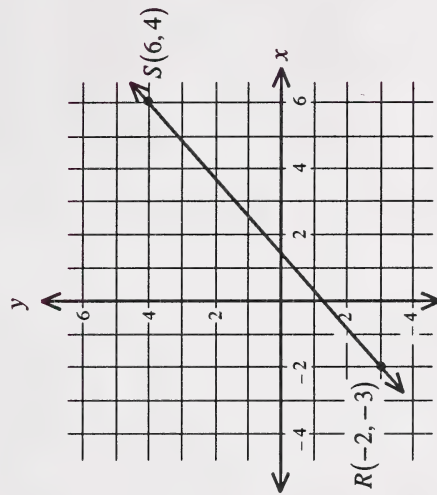


$$\begin{aligned} m_{\overline{KM}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - 3}{4 - (-3)} \\ &= \frac{-7}{7} \\ &= -1 \end{aligned}$$

The slope is -1 ; therefore, the line falls to the right.

- $R(-2, -3)$ and $S(6, 4)$

Solution:



$$\begin{aligned} m_{\overline{RS}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - (-3)}{6 - (-2)} \\ &= \frac{7}{8} \end{aligned}$$

The slope is $\frac{7}{8}$; therefore, the graph rises to the right.

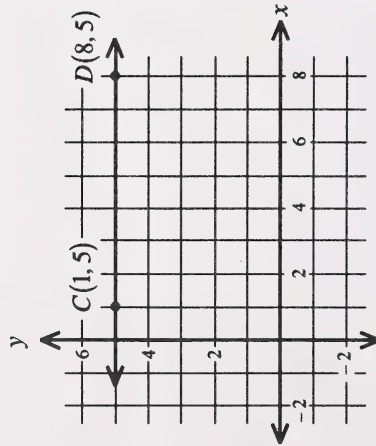
The next example shows the slopes of horizontal and vertical lines.

Example 4

Graph and find the slope of a line that contains the following points.

- $C(1, 5)$ and $D(8, 5)$

Solution:



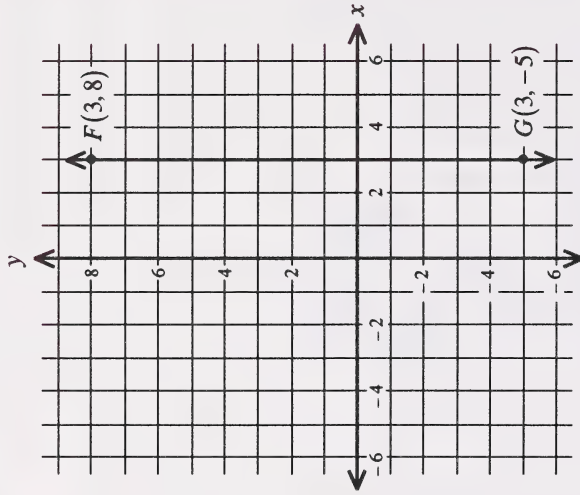
The graph of $\overleftrightarrow{CD}$ is a horizontal line through point $y = 5$. Notice that the second coordinates of each point on this line are common.

$$\begin{aligned} m_{\overleftrightarrow{CD}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 5}{8 - 1} \\ &= 0 \end{aligned}$$

The slope of a horizontal line is 0.

- $F(3, 8)$ and $G(3, -5)$.

Solution:



$$\begin{aligned} m_{\overleftrightarrow{FG}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 8}{3 - 3} \\ &= \frac{-13}{0} \\ &= \text{undefined} \end{aligned}$$

The slope of a vertical line is undefined.

$\overleftrightarrow{CD}$ is read as line CD

The horizontal line has a run, but no rise.

The vertical line has a rise, but no run.

By studying the preceding examples you can see the following:

- If a line has a positive slope ($m > 0$), the line will **rise to the right**.
- If a line has a negative slope ($m < 0$), the line will **fall to the right**.
- If a line has a slope of 1 or -1 , it runs through the coordinate plane at an angle of 45° to the horizontal axis.
- If the slope of a line is 0 ($m = 0$), the line is horizontal (parallel to the x -axis).
- If the slope of a line is undefined, the line is vertical (parallel to the y -axis).

You should be aware of these facts about the slope of a line. The next example is an interesting application of slope.

Example 5

A line passes through the points $A(k, -5)$ and $B(3, 4)$ and has a slope of $-\frac{3}{2}$. Find k .

Solution:

Find the slope of $\overleftrightarrow{AB}$.

$$\begin{aligned} m_{\overleftrightarrow{AB}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - (-5)}{3 - k} \\ &= \frac{9}{3 - k} \end{aligned}$$

You know that the slope of the line containing A and B is $-\frac{3}{2}$.

$$\begin{aligned} \frac{9}{3 - k} &= \frac{-3}{2} \\ 18 &= -9 + 3k \\ 27 &= 3k \\ k &= 9 \end{aligned}$$

Check your answer.

LS	RS
$\frac{9}{3 - k}$	$\frac{-3}{2}$
$\frac{9}{3 - 9}$	$\frac{-3}{2}$
$\frac{9}{-6}$	$\frac{-3}{2}$
$\frac{-3}{2}$	$\frac{-3}{2}$
LS	RS

Now that you are familiar with the concept of slope and its applications, try the following questions. Graph paper is provided in **Appendix B**.

Complete questions 6, 8, 9, 10a, and 11. Do the others if you feel you need more practice.

5. Graph each line segment and find its slope. State whether it rises or falls to the right.

a. $A(-2, -5), B(4, 6)$

b. $M(2, 7), N(5, -3)$

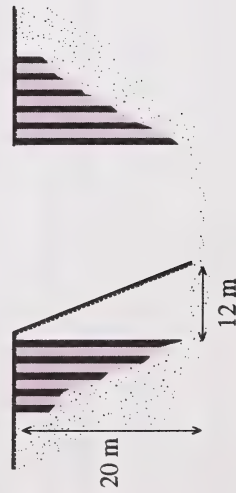
c. $P(3, -5), Q(-2, 8)$

d. $S(1, 6), T(-5, -5)$

6. $\overline{AB}$ is parallel to the y-axis. What is the slope of this line segment?

7. The vertices of a quadrilateral are $(5, 4), (-5, 5), (-5, -4)$, and $(5, -5)$. Give the slopes of the diagonals.

8. A railway bridge that once extended across a canyon collapsed. The rail track fell 20 m. Its horizontal distance from the side of the canyon is now 12 m. What is the slope of this track?



9. The top of a ladder reaches the edge of a roof. Explain how you could find the slope of the ladder.

10. Find the value of x for each line segment joining the given points with the given slope.

a. $A(3, -3), B(x, 7)$, slope $= -\frac{5}{3}$

b. $P(4, x), Q(-4, -10)$, slope $= 2$

11. Find the value of k if the slope of the line joining $(-3, k)$ and $(7, 7)$ is 0.



For solutions to Activity 2, turn to **Appendix A**, **Topic 3**.



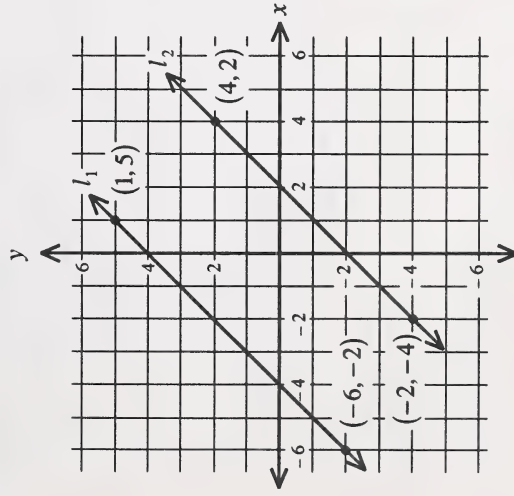
Activity 3



Identify and use the relationship between slopes of parallel lines.

How do the slopes of two parallel lines compare? In the following figure, l_1 and l_2 are two parallel lines. What is the slope of each line?

The slope formula will help you find them.



$$\begin{aligned}\text{slope of } l_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - (-2)}{1 - (-6)} \\ &= \frac{7}{7} \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{slope of } l_2 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-4)}{4 - (-2)} \\ &= \frac{6}{6} \\ &= 1\end{aligned}$$

The slopes of both lines are equal to 1. You can now conclude that parallel lines have equal slopes. This illustrates an important property.

If the lines are parallel, they have equal slopes.

If the slopes are equal, the lines are parallel.



Go to the next example and see how this property applies to a parallelogram.

Parallel lines are lines which are the same distance apart no matter how far they are extended. In other words, they will never meet.

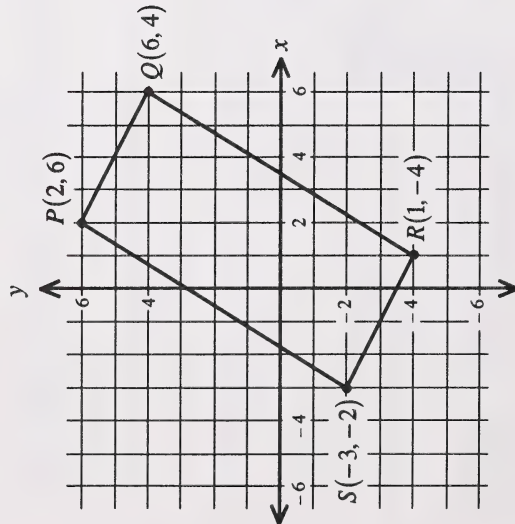
If two lines are parallel, they must have the same slope.

Example 6

Show that quadrilateral $PQRS$ is a parallelogram if its vertices are $P(2, 6)$, $Q(6, 4)$, $R(1, -4)$, and $S(-3, -2)$.

Solution:

Draw the quadrilateral on a coordinate plane. You must show that the opposite sides are parallel to each other. Find the slopes and compare them.



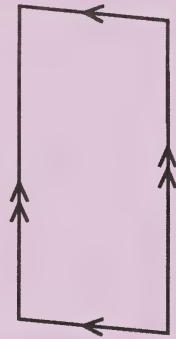
$$\begin{aligned}\text{slope of } \overline{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 6}{6 - 2} \\ &= \frac{-2}{4} \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{slope of } \overline{SR} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - (-2)}{1 - (-3)} \\ &= \frac{-2}{4} \\ &= -\frac{1}{2}\end{aligned}$$

$$m_{\overline{PQ}} = m_{\overline{SR}}$$

$$\therefore \overline{PQ} \parallel \overline{SR}$$

When the opposite sides of a quadrilateral are parallel, it is a parallelogram. The following is an example.



A diagram will help you to confirm your calculation and give you an idea of what the figure will look like.

The symbol $\parallel$ means parallel.

$$\begin{aligned}\text{slope of } \overline{PS} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 6}{-3 - 2} \\ &= \frac{-8}{-5} \\ &= \frac{8}{5}\end{aligned}$$

$$\begin{aligned}\text{slope of } \overline{QR} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - 4}{1 - 6} \\ &= \frac{-8}{-5} \\ &= \frac{8}{5}\end{aligned}$$

$$m_{\overline{PS}} = m_{\overline{QR}}$$

$$\therefore \overline{PS} \parallel \overline{QR}$$

Since the opposite sides of the quadrilateral are parallel, $PQRS$ is a parallelogram.

Try the following questions.

Do at least two parts from question 1. Then complete questions 3, 4, 5, and 6. Do the others if you need more practice.

1. The slopes of two lines are given. State whether they are parallel.

a. $m_1 = \frac{2}{3}$ and $m_2 = \frac{10}{15}$

b. $m_1 = 3\frac{4}{8}$ and $m_2 = 3.5$

c. $m_1 = 1$ and $m_2 = -1$

d. $m_1 = \frac{2}{7}$ and $m_2 = \frac{7}{2}$

2. Which of the line segments are parallel?

a. $(9, 7)$, $(8, 1)$ and $(-3, 1)$, $(4, 7)$

b. $(8, -3)$, $(-1, 5)$ and $(-11, 13)$, $(-2, 5)$

3. The line through $A(6, p)$ and $B(5, 3)$ is parallel to the line through $E(0, -4)$ and $F(6, 10)$. Find the value of p .

4. A triangle has vertices $A(8, 2)$, $B(-6, -2)$, and $C(4, -6)$. Use the slope formula to show that the line through the midpoints of $\overline{AB}$ and $\overline{AC}$ is parallel to $\overline{BC}$.

5. The slopes of two parallel lines are $-\frac{3}{7}$ and $\frac{6}{m}$. Find the value of m .

6. Why is it helpful to have numbered streets and avenues parallel to each other in a town?



For solutions to Activity 3, turn to **Appendix A, Topic 3**.

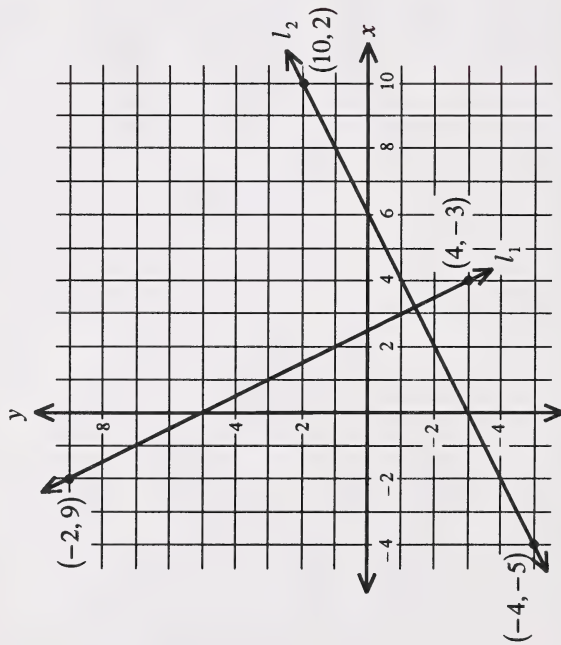
Activity 4



Identify and use the relationship between slopes of perpendicular lines.

Can you determine whether two lines are perpendicular if you know their slopes? To find the slopes of perpendicular lines, use the same formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$.

The following diagram shows two perpendicular lines. Compare the slopes of these two lines. It is convenient to use m_1 to refer to the slope of l_1 and m_2 to refer to the slope of l_2 .



Perpendicular lines are lines that intersect at 90° . Thus, they form a right angle.

$$\begin{aligned}
 m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{9 - (-3)}{-2 - 4} \\
 &= \frac{12}{-6} \\
 &= -2
 \end{aligned}$$

$$\begin{aligned}
 m_2 &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{2 - (-5)}{10 - (-4)} \\
 &= \frac{7}{14} \\
 &= \frac{1}{2}
 \end{aligned}$$

Notice that one slope is positive ($\frac{1}{2}$) and the other is negative (-2).

Notice further that if you disregard the signs, one slope is the reciprocal of the other. In other words, 2 is the reciprocal of $\frac{1}{2}$ and $\frac{1}{2}$ is the reciprocal of 2.

Therefore, the slope of one line is equal to the negative reciprocal of the slope of the other line.

In symbol form, this can be expressed as follows:

$$m_2 = -\frac{1}{m_1}$$

The negative reciprocal of -2 is $\frac{1}{2}$.

The negative reciprocal of $\frac{1}{2}$ is -2 .

This demonstrates an important property for perpendicular lines.

If two lines are perpendicular to each other, they have slopes that are negative reciprocals.

OR



If the slopes of two lines are negative reciprocals, the two lines are perpendicular to each other.

You may also observe that if two lines are perpendicular, the product of their slopes is -1 .

Recall the reciprocal relationship.

$$\text{number} \times \text{reciprocal} = 1$$

$$x \times \frac{1}{x} = 1$$

If $m_1 \times m_2 = -1$, the lines are perpendicular.

For the perpendicular lines l_1 and l_2 , $-2 \times \frac{1}{2} = -1$.

The idea of negative reciprocals is applied in the next example.

Example 7

The equation of a line is $2x + 3y = 6$. Show that this line is perpendicular to a line segment whose endpoints are $F(4, -12)$ and $G(8, -6)$.

Solution:

You have to compare the slope of $2x + 3y = 6$ with the slope of $\overline{FG}$.

$$\begin{aligned} m_{\overline{FG}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-6 - (-12)}{8 - 4} \\ &= \frac{6}{4} \\ &= \frac{3}{2} \end{aligned}$$

If you know two points on the line defined by $2x + 3y = 6$, you can calculate its slope. Two points can be located using the x - and y -intercepts.

The x -intercept is the point where the line crosses the x -axis; therefore, at that point, $y = 0$.

To find the x -intercept for $2x + 3y = 6$, let $y = 0$.

$$2x + 3(0) = 6$$

$$2x = 6$$

$$x = 3$$

Therefore, the x -intercept is $(3, 0)$.

The y -intercept is the point where the line crosses the y -axis; therefore, at that point, $x = 0$.

To find the y -intercept for $2x + 3y = 6$, let $x = 0$.

$$2(0) + 3y = 6$$

$$3y = 6$$

$$y = 2$$

Therefore, the y -intercept is $(0, 2)$.

Now that you know two points on the line, you can calculate the slope of $2x + 3y = 6$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - 2}{3 - 0} \\ &= \frac{-2}{3} \end{aligned}$$

Now you can answer the question. Is the line $2x + 3y = 6$ perpendicular to $\overline{FG}$?

$$m_{\overline{FG}} = \frac{3}{2} \qquad m_1 = \frac{-2}{3}$$

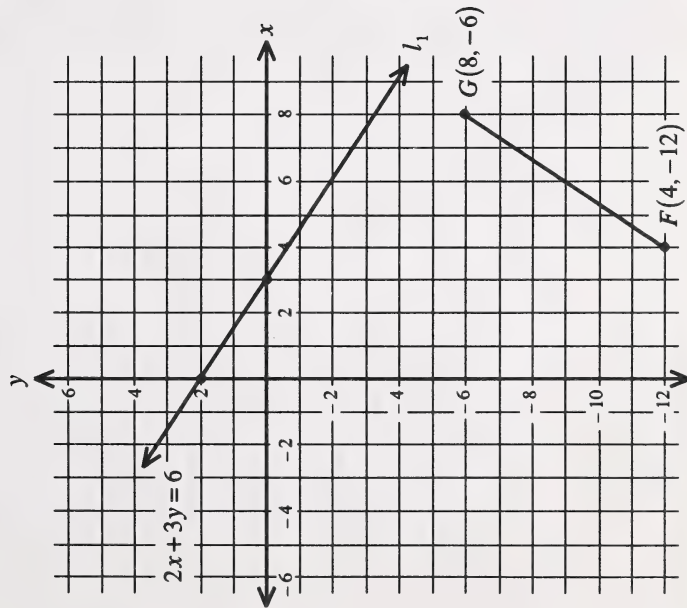
If l_1 is perpendicular to $\overline{FG}$, then the product of the slopes should be -1 .

$$\begin{aligned} m_{\overline{FG}} \times m_1 &= \frac{3}{2} \times \frac{-2}{3} \\ &= \frac{-6}{6} \\ &= -1 \end{aligned}$$

$$\therefore l_1 \perp \overline{FG}$$

The symbol $\perp$ means perpendicular.

The following example deals with a right triangle. Remember that a right triangle has a 90° angle. In other words, it has two perpendicular segments.

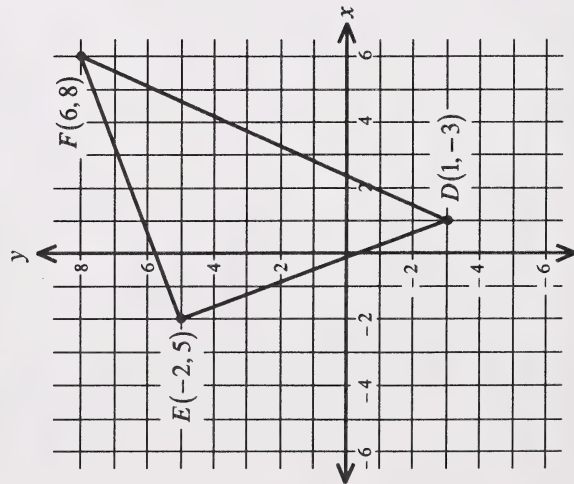


Example 8

Show that triangle DEF is a right triangle if its vertices are $D(1, -3)$, $E(-2, 5)$, and $F(6, 8)$.

Solution:

Draw a diagram. This will give you an idea of which line segments may be perpendicular to each other. According to the diagram, $\overline{EF}$ appears to be perpendicular to $\overline{DE}$. Find the slopes of these segments to confirm your belief.



$$\begin{aligned}\text{slope of } \overline{EF} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - 5}{6 - (-2)} \\ &= \frac{3}{8}\end{aligned}$$

$$\begin{aligned}\text{slope of } \overline{DE} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - (-3)}{-2 - 1} \\ &= \frac{8}{-3} \\ &= -\frac{8}{3}\end{aligned}$$

The slope of $\overline{DE}$ is the negative reciprocal of the slope of $\overline{EF}$, or $\frac{3}{8} \times -\frac{8}{3} = -1$.

Therefore, $\overline{EF} \perp \overline{DE}$ and DEF is a right triangle.

See the **Extensions** section at the end of this topic if you wish to see an alternate development.

To show that $\triangle DEF$ is a right triangle, you can also use the distance formula and the Pythagorean theorem.

Example 9

The slopes of a pair of lines are given by $3k$ and $\frac{1}{2}$. If the lines are perpendicular, find the value of k .

Solution:

If the two lines are perpendicular, the product of their slopes must equal negative one.

$$3k \times \frac{1}{2} = -1$$

$$3k = -2$$

$$k = \frac{-2}{3}$$

Now it is your turn. Try the following questions.

Do at least questions 1c, 1d, 2, 4, and 5.

1. In each of the following parts, m_1 and m_2 are the slopes of two lines. State whether the lines are parallel, perpendicular, or neither.

a. $m_1 = \frac{4}{7}$ and $m_2 = \frac{7}{4}$

b. $m_1 = \frac{-3}{2}$ and $m_2 = \frac{3}{2}$

c. $m_1 = \frac{-5}{6}$ and $m_2 = \frac{-15}{18}$

d. $m_1 = \text{undefined}$ and $m_2 = 0$

2. The slopes of two perpendicular lines are $3k$ and $\frac{-5}{6}$. Find the value of k .
3. Two lines are parallel. If their slopes are $(4p+7)$ and 5, find the value of p .

The negative reciprocal of $\frac{1}{2}$ is -2 .

A vertical line is perpendicular to a horizontal line.

Activity 5



Identify collinear points and use slope to determine if points in a plane are collinear.

4. The vertices of a quadrilateral are $A(-5, 5)$, $B(10, 4)$, $C(5, -5)$ and $D(-10, -4)$.

- Show that $ABCD$ is a parallelogram.
- Find the slopes of the diagonals.

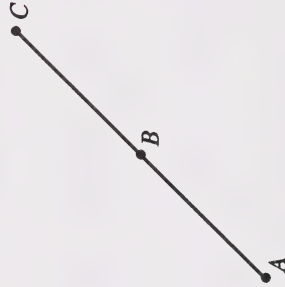
5. An isosceles triangle DEF has the vertices $D(-2, 5)$, $E(-9, -4)$, and $F(5, -4)$. Points G and H are the midpoints of $\overline{DE}$ and $\overline{DF}$ respectively.

- Show that $\overline{GH}$ is parallel to $\overline{EF}$.
- $\overline{DK}$ is a median. Show that $\overline{DK}$ is perpendicular to $\overline{EF}$.

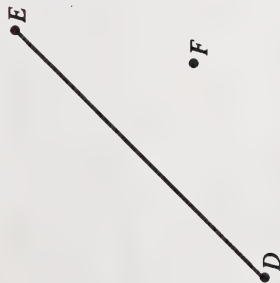


For solutions to Activity 4, turn to **Appendix A, Topic 3**.

Remember: A median of a triangle is a segment joining a vertex of the triangle and the midpoint of the opposite side.



These points are collinear.



These points are noncollinear.

From the previous diagrams, you can say the following:

If the slope of $\overline{AB}$ is equal to the slope of $\overline{BC}$, then points A , B , and C are collinear.



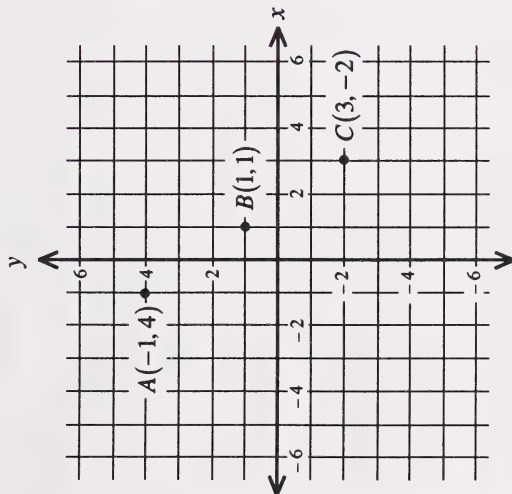
Collinear points are points that lie on the same straight line.

Look at the following example to see a proof for collinear points.

Example 10

The points $A(-1, 4)$, $B(1, 1)$, and $C(3, -2)$ lie in a plane. Show that A , B , and C are collinear.

Solution:



Remember: Three or more points are collinear if they lie on the same line.

If $m_{\overline{AB}} = m_{\overline{BC}}$, then A , B , and C are collinear. (m represents the slope.)

Show that the slope of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are equal.

$$\begin{aligned} m_{\overline{AB}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 4}{1 - (-1)} \\ &= \frac{-3}{2} \end{aligned}$$

$$\begin{aligned} m_{\overline{BC}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 1}{3 - 1} \\ &= \frac{-3}{2} \end{aligned}$$

$$\begin{aligned} m_{\overline{AC}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 4}{3 - (-1)} \\ &= \frac{-6}{4} \\ &= \frac{-3}{2} \end{aligned}$$

Since the slopes of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are equal and the points A, B, and C lie in a plane, then A, B, and C are collinear.

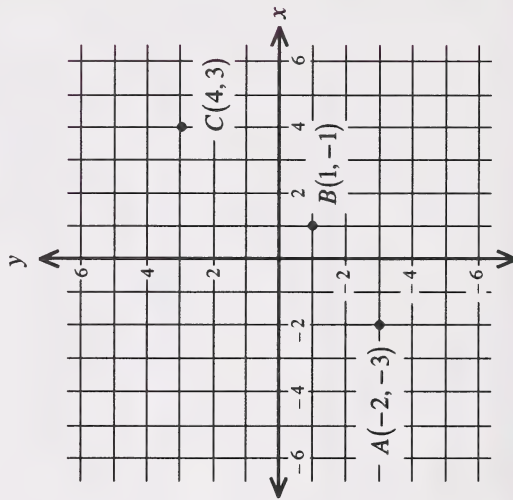
In the previous example, the slope was found for three pairs of points ($\overline{AB}$, $\overline{BC}$, and $\overline{AC}$). An error in the calculations would be indicated if only two of the slopes had the same value. If this occurs, go back to all three calculations and make the appropriate corrections.

The following is an example that demonstrates three points that are noncollinear.

Example 11

Determine whether A(-2, -3), B(1, -1), and C(4, 3) are collinear.

Solution:



Note: If the slopes of any two segments with a common endpoint are the same, the three points lie on the same line.

Find the slope of $\overline{AB}$.

$$m_{\overline{AB}} = \frac{(-1) - (-3)}{1 - (-2)} \\ = \frac{2}{3}$$

Find the slope of $\overline{BC}$.

$$m_{\overline{BC}} = \frac{3 - (-1)}{4 - 1} \\ = \frac{4}{3}$$

Find the slope of $\overline{AC}$.

$$m_{\overline{AC}} = \frac{3 - (-3)}{4 - (-2)} \\ = \frac{6}{6} \\ = 1$$

The slopes of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are not equal. Therefore, the points are noncollinear.

Notice that the slopes for the three pairs of points are all different.

Now try the following questions.

Do either question 1 or question 2. Complete questions 3 and 4.

- Show that the points are collinear.
 - $D(5, 1)$, $E(3, 2)$, $F(-5, 6)$
 - $P(8, -3)$, $Q(4, 2)$, $R(-2, 9.5)$
- If $A(4, 200)$, $B(2, 100)$, and $C(k, 150)$ are collinear, find the value of k .
- The points $R(4, -4)$, $M(2, -1)$, and $S(-2, 5)$ lie in a plane. One method to prove that R , M , and S are collinear is to use the slope method. Can you think of another method? Explain.
- Susan is flying from Parksville to Forestview. The coordinates of Parksville are $(3, 7)$, and the coordinates of Forestview are $(5, -5)$. Is Aquacity on the flight path between Parksville and Forestview, if the coordinates of Aquacity are $(4, 1)$.



For solutions to **Activity 5**, turn to **Appendix A, Topic 3**.

When the slopes of three pairs of points are all equal, the points are collinear. When the slopes of three pairs of points are all different, the points are noncollinear.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

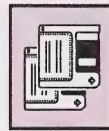
} You may decide to do both.



Extra Help

Do Part A or Part B.

Part A



Use the computer program entitled Algebra Drill and Practice,¹ disk 1, part 3 on slope and equations of lines. Do only section 1 on slope given two points. Do as many questions as you feel are necessary.

Part B



Do at least the odd-numbered questions.

¹ Algebra Drill and Practice is a title of *Conduit*.

1. Fill each blank with the correct answer.

- a. In the relationship $m = \frac{\text{rise}}{\text{run}}$, rise represents the _____ height and run represents the _____ distance.
- b. If $m = \frac{9}{11}$, the vertical height is _____.
- c. If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a line, then $(x_2 - x_1)$ represents the _____ in the slope formula.
- d. If the slope is negative, the line _____ to the right.
- e. The slope of a vertical line is _____.
- f. The slope of a horizontal line is _____.
- g. If two lines are perpendicular, the slope of one line will be the _____ of the slope of the other line.

2. Why is the slope of a vertical line undefined?

3. Find the rise when two points on a line are $P_1(-2, 5)$ and $P_2(4, -11)$.

4. Find the slope of the line through the points $A(-6, -7)$ and $B(4, -15)$. Does the line rise or fall to the right?

5. Points $P(4, 1)$, $Q(1, 3)$, and $R(3, 6)$ are the vertices of a triangle. Show that Q is a right angle.



For solutions to **Extra Help**, turn to **Appendix A, Topic 3**.



Extensions

Another way of finding the slope of a line from its equation is to assign values to x and calculate the corresponding values of y . You need at least two sets of ordered pairs to find the slope of the line. However, more work is involved in this method.

Use the same equation that is given in Example 7 so you can see the comparison.

$$2x + 3y = 6$$

You can choose any suitable value for x .

Substitute $x = 1$.

$$2(1) + 3y = 6$$

$$2 + 3y = 6$$

$$3y = 6 - 2$$

$$3y = 4$$

$$y = \frac{4}{3}$$

The ordered pair is $(1, \frac{4}{3})$.

Substitute $x = -1$.

$$2(-1) + 3y = 6$$

$$-2 + 3y = 6$$

$$3y = 6 + 2$$

$$3y = 8$$

$$y = \frac{8}{3}$$

The ordered pair is $(-1, \frac{8}{3})$.

Now use the two ordered pairs in the slope formula.

Let $(x_1, y_1) = (1, \frac{4}{3})$ and $(x_2, y_2) = (-1, \frac{8}{3})$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{\frac{8}{3} - \frac{4}{3}}{-1 - 1} \\ &= \frac{\frac{4}{3}}{-2} \\ &= \frac{4}{3} \times -\frac{1}{2} \\ &= -\frac{2}{3} \end{aligned}$$

The slope is $-\frac{2}{3}$.

Now do the following question to see if you understand this method. Complete either 1a or 1b.

1. Use the method demonstrated in this section to calculate the slope of the following.

a. $x - 2y = 4$

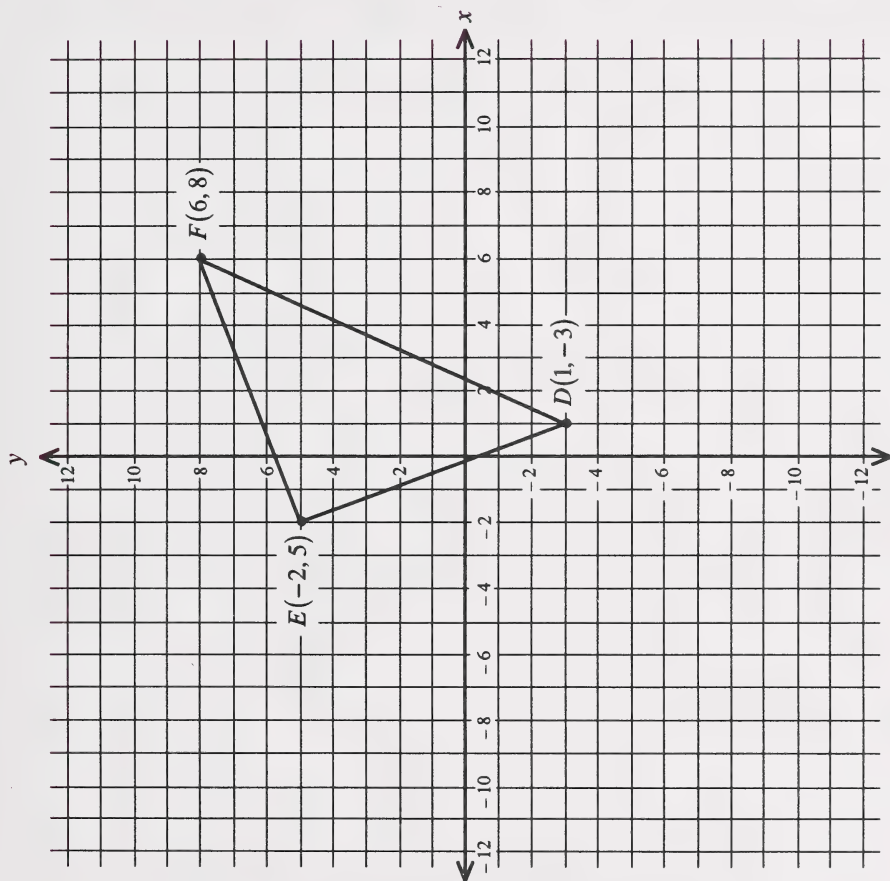
b. $2y - 4x = 0$



For solutions to **Extensions**, turn to **Appendix A, Topic 3**.

An alternate method to Example 8 is as follows.

The vertices of $\triangle DEF$ are $D(1, -3)$, $E(-2, 5)$, and $F(6, 8)$. Use the distance formula and the Pythagorean theorem to prove that $\triangle DEF$ is a right triangle. You have to find the length of each side using its endpoints.



The distance formula is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

By substituting $(1, -3)$ and $(-2, 5)$, the length of $\overline{DE}$ is as follows:

$$\begin{aligned} DE &= \sqrt{(-2 - 1)^2 + [5 - (-3)]^2} \\ &= \sqrt{(-3)^2 + (8)^2} \\ &= \sqrt{9 + 64} \\ &= \sqrt{73} \end{aligned}$$

By substituting $(-2, 5)$ and $(6, 8)$, the length of $\overline{EF}$ is as follows.

$$\begin{aligned} EF &= \sqrt{[6 - (-2)]^2 + (8 - 5)^2} \\ &= \sqrt{(8)^2 + (3)^2} \\ &= \sqrt{64 + 9} \\ &= \sqrt{73} \end{aligned}$$

By substituting $(1, -3)$ and $(6, 8)$, the length of $\overline{DF}$ is as follows:

$$\begin{aligned} DF &= \sqrt{(6 - 1)^2 + [8 - (-3)]^2} \\ &= \sqrt{(5)^2 + (11)^2} \\ &= \sqrt{25 + 121} \\ &= \sqrt{146} \end{aligned}$$

Then the Pythagorean theorem can be applied.

LS	RS
$(DF)^2$	$(DE)^2 + (EF)^2$
$(\sqrt{146})^2$	$(\sqrt{73})^2 + (\sqrt{73})^2$
146	$73 + 73$
146	146
LS	RS

Therefore, $\triangle DEF$ is a right triangle.

Now try the following question.

- Use the distance formula to show that ABC is a right triangle if its vertices are $A(10, 2)$, $B(-1, -9)$, and $C(4, 8)$.



For solutions to **Extensions**, turn to **Appendix A, Topic 3**.

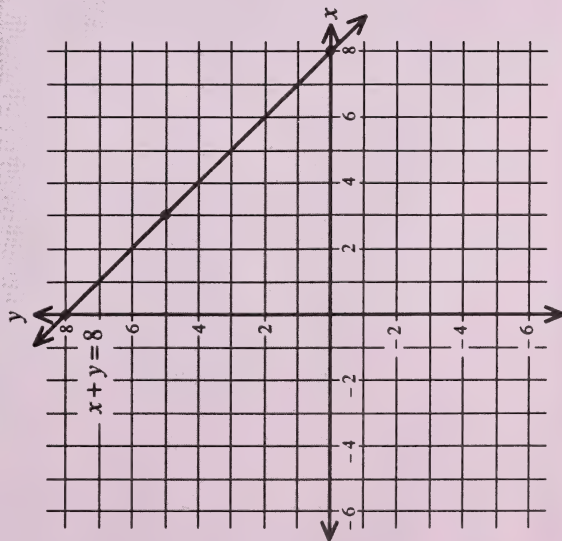
The calculations show that $\overline{DF}$ is the longest segment. Since $\angle E$ is opposite $\overline{DF}$, it is the 90° angle.

Topic 4 Graphing Linear Equations



Introduction

In this topic you will spend some time reviewing terminology related to graphing; then you will go into working with the graphs of equations of lines.



What Lies Ahead

Throughout the topic you will learn to

1. use terminology and graphing skills related to the graphing of ordered pairs
2. develop a table of values, and sketch the graph of a linear equation
3. determine x - and y -intercepts

Now that you know what to expect, turn the page to begin your study of graphing linear equations.



Exploring Topic 4

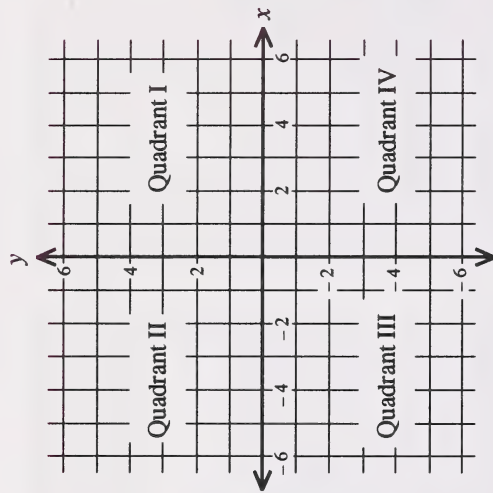
Activity 1



Use terminology and graphing skills related to the graphing of ordered pairs.

The Cartesian plane or coordinate plane is formed by two intersecting number lines called the x - and y -axes. These axes are perpendicular (at right angles), and the point where they meet is called the origin. This is represented by $(0, 0)$.

The x - and y -axes divide the coordinate plane into four regions called quadrants. Refer to the following diagram.

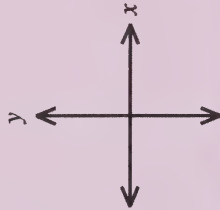


Now that you know how the coordinate plane is designed, plot some points on it.

Look at the following example.



Note: The y -axis is always vertical. The x -axis is always horizontal.

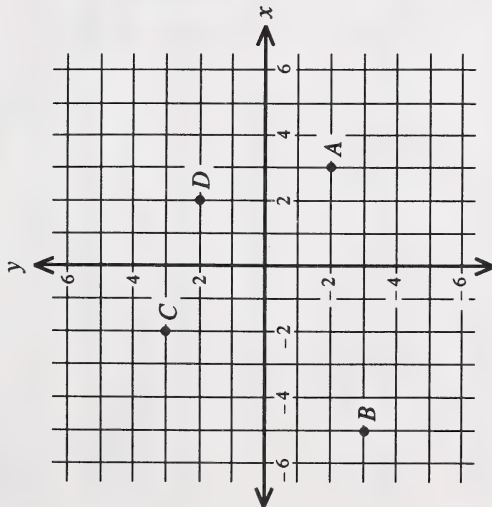


Note: Any point located on either the x - or y -axis is not considered to be in any quadrant.

Example 1

Plot $A(3, -2)$, $B(-5, -3)$, $C(-2, 3)$, and $D(2, 2)$ on a coordinate plane.

Solution:



Notice that there is no particular pattern among these points. However, in many other instances, the points will have a distinct pattern which you will investigate in the next activity.

1. In which quadrant will you find the following points?

a. $(3, 8)$

b. $(-1, -2)$

c. $(0, 3)$

2. What can you conclude about any point in Quadrant IV?



For solutions to **Activity 1**, turn to **Appendix A, Topic 4**.

A graphing calculator or a computer graphing program may be used to plot points and graph linear equations. If you have access to either technology, use it to do the graphing calculator sections of this unit indicated by the



graphing calculator icon.

You may also use the graphing calculator to check the graphs that have been used in the print part of this unit.

To plot any ordered pair, the order is important. To plot the ordered pair $(3, -2)$, start at the origin and move three units to the right along the x -axis. From this point, move two units down parallel to the y -axis. You are now at the point $(3, -2)$.

There are a number of graphing calculators and computer graphing programs available on the market today. This course will discuss two graphing calculators: the CASIO *fx-7700G* and the Texas Instruments TI-81. Instructions on how to use the graphing calculator to graph linear equations are given. If you have difficulty, you should consult the manual that came with your calculator. Two computer programs that may be used to graph linear equations are Zap-a-Graph and Master Grapher. Zap-a-Graph is for the Macintosh only while Master Grapher is available for Commodore, IBM, or Macintosh. If you have access to any of these computer programs, follow the instructions that came with the program carefully.

Setting the range

To use a graphing calculator, a range that is appropriate to the problem is required. The following instructions show how to set the range. Whenever you see a function name enclosed in a box, you are to press that particular key on the calculator.

CASIO *fx-7700G* or similar Casio Graphing Calculators

Turn on the calculator by pressing **ON**. To set the range, press **RANGE** and type in the desired value at the flashing cursor. You will want to enter six values one for each Xmin, Xmax, Xscl (scale), Ymin, Ymax, and Yscl (scale). Press **EXE** (execute) after each entry.

A good set of values to enter for the range is $-10, 10, 1, -10, 10, 1$. To enter a negative number you may use the negative sign (**SHIFT** **Ans**) or use the minus sign. Some calculators will allow you to use only the negative sign for negative numbers.

After you have entered the six values, press **EXE** three more times to go back to the screen that appeared when you first turned the calculator on.

At times you may have to enter a different range such as 0, 10, 1, 0, 120, 20. In this case the numbers on the x -axis increase by 1 unit from 0 to 10, and the numbers on the y -axis increase by 20 units from 0 to 120.

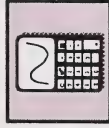
TI-81

Setting the range on the TI-81 is very similar to setting the range on the CASIO *fx-7700G*. Turn on the calculator by pressing **ON**.

Press **RANGE** and enter the six values, one for each of Xmin, Xmax, Xscl (scale), Ymin, Ymax, and Yscl (scale). Press **ENTER** after each entry.

A good set of values to enter for the range is $-10, 10, 1, -10, 10, 1$. On the TI-81 you must use the negative sign ($-$) not the subtraction sign (the one without the brackets).

After you have entered the six values, press **Y=**. Now you are ready to enter an equation. More instructions on entering points and equations will be given with each exercise.



The graphing calculator can be used for a simple task such as plotting points. To plot a point, use the PLOT function.

To plot a point using a CASIO fx-7700G graphing calculator, follow these steps:

Step 1: Turn the graphing calculator on. Press **SHIFT**, **F5**, and **EXE** to clear the graphing screen. Set the range to -10 to $+10$.

Step 2: Press **SHIFT** and then **F3**. Plot should appear on the screen.

Step 3: Enter the first coordinate. (You may use the negative sign or the minus sign if the number is negative.)

Step 4: Press **SHIFT** and **→** to get a comma.

Step 5: Enter the second coordinate.

Step 6: Press **EXE**.

Step 7: To plot another point, press **SHIFT** and **F3**, and repeat Step 3 to Step 6.

To plot a point using the TI-81 graphing calculator, follow these steps:

Step 1: Turn the graphing calculator on. Press **Y=** and **CLEAR** for each equation to clear the graphing screen. Set the range to -10 to $+10$.

Step 2: Press **2nd** and **PRGM** to get the DRAW function.

Step 3: Move down to PT-On (number 3) using the down arrow **▼**, and press enter **ENTER**.

Step 4: PT-On (should appear on the screen.

Step 5: Enter the first coordinate.

Step 6: Press **ALPHA** and **.** to get a comma.

Step 7: Enter the second coordinate. Press **)** to enclose the coordinate.

Step 8: Press **ENTER**. A point should appear on the graph.

Step 9: Press **CLEAR** and repeat Step 2 to Step 8 to draw any additional point.

To clear all the points on the screen, go to the DRAW function and press **ENTER** twice.

Plot the following points using a graphing calculator or a computer graphing program. Which quadrant is each point located?

3. $(5, 3)$
4. $(-4, -1)$
5. $(-2, 6)$
6. $(3, -3)$



For solutions to **Activity 1**, turn to **Appendix A, Topic 4**.



Activity 2



Develop a table of values, and sketch the graph of a linear equation.

In the next example, notice the direct relation between the hours worked x and the dollars earned y .

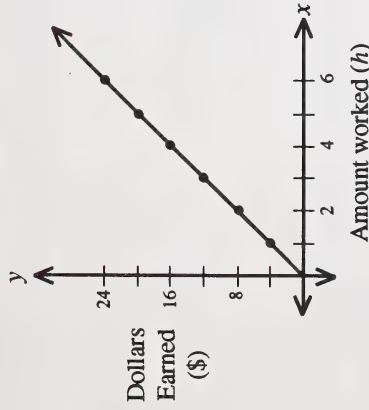
Example 2

Jason works at Super Foods Restaurant and earns \$4.00 per hour. Construct a table that shows the amount he earned after each hour, and graph this relation.

Solution:

hours x	dollars y
0	0
1	4
2	8
3	12
4	16
5	20
6	24

The table is referred to as a table of values. This shows a correspondence between the two variables. The amount that Jason earned depends upon the number of hours worked.



After 0 h, Jason earned \$0; after 1 h Jason earned \$4; and after 2 h he earned \$8. These correspond to the ordered pairs $(0, 0)$, $(1, 4)$, and $(2, 8)$.

Since Jason might work a portion of an hour, for example $3\frac{1}{2}$, you can add the point generated. Also, extend the line with an arrow because he may work longer than 6 h.

The ordered pair (x, y) satisfies the equation $y = 4x$. Since the graph is a straight line, this is an example of a linear equation in two variables.



The purpose of the equation is to relate all ordered pairs on the line. Ordered pairs that satisfy the equation are on the line.

To graph any linear equation, do the following:

- Construct a table of values.
- Plot the corresponding pairs on the graph.
- Join the points.

Choose a value for x . The equation will then determine the corresponding value for y . Work through the following example.

Example 3

Make a table of values for $y = 3x + 2$, and draw the graph.

Solution:

First, choose the x -values that you would like to use. Integers are convenient to work with.

x	y
-3	
0	
3	

Note: The x - and y -axes have different scales, in order to be able to easily plot the large y -values in the space allowed on this page. Usually the scales of the x - and y -axes are equal.

You can choose any value for x . In **Unit 1**, you saw that every real number can be associated with a point on a continuous line.

It is safer to choose at least three values for x . The third point serves as a check in case there is an error in your calculations.

Now find the y -values associated with your chosen x -values.

$$\begin{aligned}\text{When } x = -3, y &= 3(-3) + 2 \\ &= -9 + 2 \\ &= -7\end{aligned}$$

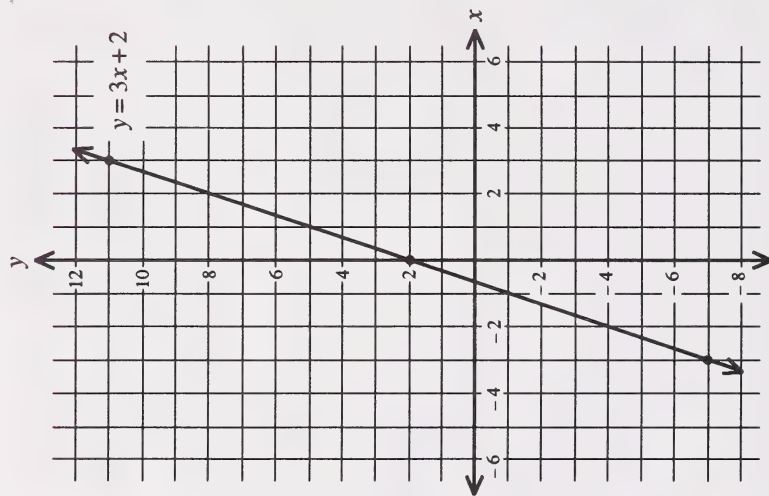
$$\begin{aligned}\text{When } x = 0, y &= 3(0) + 2 \\ &= 0 + 2 \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{When } x = 3, y &= 3(3) + 2 \\ &= 9 + 2 \\ &= 11\end{aligned}$$

The table of values is as follows:

x	y
-3	-7
0	2
3	11

Graph.



Study the next example.

To determine these, substitute the x -values into the equation, and calculate the y -values.

Example 4

A television repair technician charges a flat fee of \$10 plus \$25 per hour for labour.

- Set up an equation showing the relationship between the cost and the number of hours. Then, draw the graph.

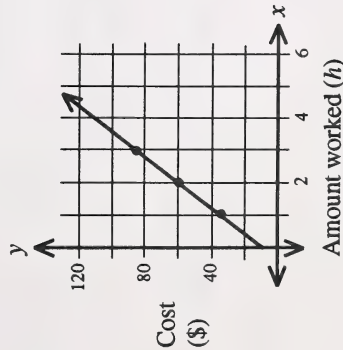
Solution:

Let y be the cost and x the the number of hours.

The equation is $y = 10 + 25x$.

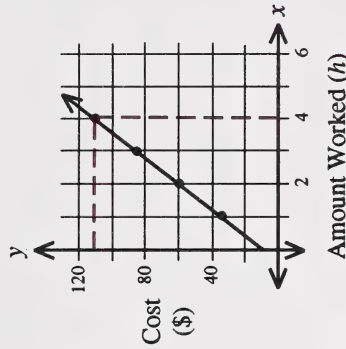
Construct a table of values and the graph.

x	y
1	35
2	60
3	85



- From the graph find the cost of repairing a television set that took four hours.

Solution:



The cost is approximately \$110.

- Does the value of one variable depend on the other.

Solution:

Yes, the value of one variable depends on the other.

- Are there any restrictions on the values to be considered?

Solution:

Yes, you cannot have negative values for x or y . You cannot work negative hours.

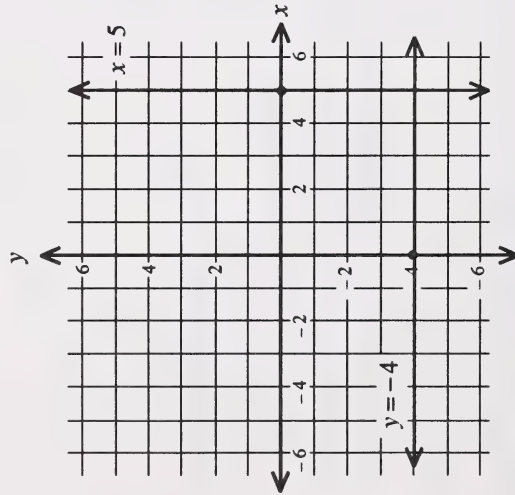
Example 5

Draw the graphs for $x = 5$ and $y = -4$.

Solution:

The graph of $x = 5$ defines a vertical line which crosses the x -axis at 5. The value of y has no effect.

The graph of $y = -4$ defines a horizontal line which crosses the y -axis at -4 . The value of x has no effect.



Try the following exercises. Answer at least questions 1b, 2, 3, and 4a or 4b. Graph paper is provided in **Appendix B**.

Do the others if you need more practice.

- Complete the following table of values, and graph the equation.

a. $y = -x + 4$ b. $2x - 3y = 6$

x	y
3	1
-2	
1	
4	

x	y
-6	-6
-3	
3	
6	

- Colleen works for a computer firm and earns \$10.00 per hour.

- Write an equation showing Colleen's earnings.
- Construct a table of values, and graph the points.
- Is it acceptable to join all the points? Why?
- Can the graph be extended in both directions? Explain.
- How many hours does Colleen have to work to make \$100?

3. A printing firm charges a standard fee of \$50 plus \$25 for each textbook printed.

a. Write an equation using x and y , where x represents the number of textbooks and y is the cost.

b. Construct a table of values, and graph the points.

c. Why does the graph begin at 50?

d. Are there any restrictions to be considered? Explain.

e. From the graph, find the cost of printing eight textbooks.

4. Graph the following.

a. $x = -4$

b. $y = 3$



For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.



You can also graph linear equations using a graphing calculator. The graphing calculator requires the equation to be in $y = mx + b$ form. Refer to **Topic 5**, activities 1 and 2 to review equations in $y = mx + b$ form. You will have to change all the equations to this form before you enter them. Use the equation in Example 6 of this topic to check if you are entering equations properly.

For the CASIO fx-7700G, follow these steps:

Step 1: Turn the graphing calculator on.

Step 2: Set the range to -10 to $+10$.

Step 3: Press **SHIFT**, **F5**, and **EXE** to clear any previous graphs.

Step 4: Press **GRAPH**. Graph $Y=$ should appear on the screen.

Step 5: Enter the equations using the numbers, and press **X, θ , T**.

This key will give you the variable X . You can also press **ALPHA** and **+** to get the variable X .

Step 6: Press **EXE**.

Step 7: To graph another equation on the same axes, repeat Step 4 to Step 5. To graph another equation on a new set of axes, repeat Step 3 to Step 5.

On the CASIO fx-7700G, you may draw as many graphs as you want on one set of axes (until you run out of memory).

For the TI-81, follow these steps;

Step 1: Turn the graphing calculator on.

Step 2: Press **Y=**. The following should appear on the screen.

:Y1=
:Y2=
:Y3=
:Y4=

Step 4: Press **GRAPH**.

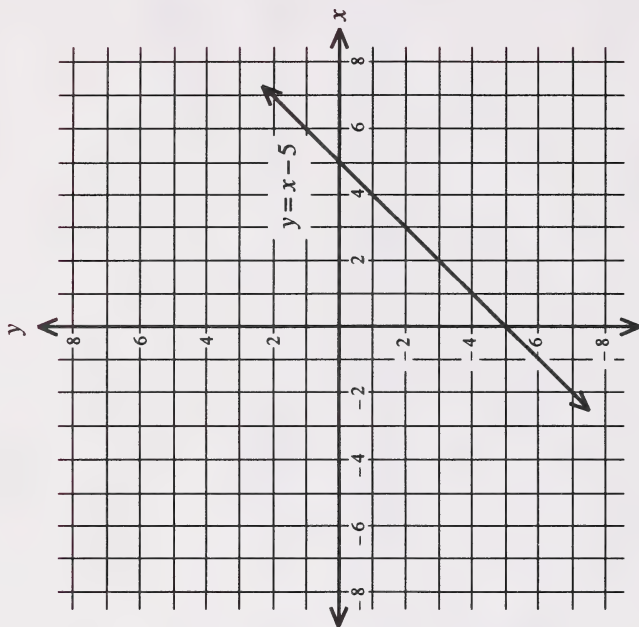
Step 5: To draw another graph on a new set of axes, place the cursor on the equation in :Y1= and press **CLEAR**. Then enter the new equation in :Y1= and press **GRAPH**. To draw another graph on the same set of axes, press **Y=** and enter the new equation in :Y2=.

On the TI-81, you may draw up to four graphs on one set of axes.

Example 6

Graph $y = x - 5$.

Solution:



Example 7

A VCR repair technician charges a flat fee of \$25 plus \$15 per hour of labour.

- Set up an equation showing the relationship between the cost and the number of hours worked. Draw a graph using a graphing calculator.

Solution:

Let y be the cost and x the number of hours worked.

The equation is $y = 25 + 15x$.

To graph the equation on your calculator, you must first set the range. Set $X_{\min} = 0$, $X_{\max} = 10$, $X_{\text{scl}} = 1$, $Y_{\min} = 0$, $Y_{\max} = 120$, and $Y_{\text{scl}} = 20$.

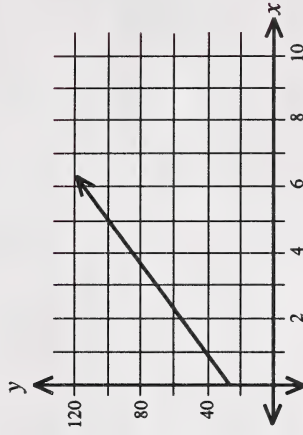
Negative values are not used since the technician does not work less than zero hours.

A maximum of 10 h on the x -axis is sufficient for this question. A maximum of \$120 on the y -axis is sufficient since $(15 \times 3) + 25 = 70$. This falls within the range. These ranges can be extended or reduced for a different question.

A scale of 20 on the y -axis is good since 120 divided by 20 is 6. The standard scale gives ten increments on the axes. A scale of 10 can be used as well. This will give twelve increments on the y -axis.

Now enter the equation. Follow the instructions given previously to enter the equation.

The graph should look as follows:



- What is the cost if the repairs took 3 h?

Solution:

Use the TRACE and ZOOM functions to find the cost.

To use TRACE on the CASIO fx-7700G, follow these steps:

Step 1: Press **F1** and use **▶** or **◀** to move the cursor.

Step 2: When you are at the point where the x -value is as close to 3 as possible, press **ZOOM**. Five functions will appear at the bottom of the screen.

Step 3: Press **F1** for BOX; then, move the cursor close to where the line crosses $x = 3$. Press **EXE**. This will set a starting point.

Step 4: Now move the cursor in any direction in order to go past the point. Notice that a line is drawn as you move the cursor. When you are past the point $x = 3$, move the cursor away from the line past the point again. This time you should see a box drawn.

Step 5: Once the point is enclosed in the box, press **EXE**. The section that was in the box will be drawn and expanded.

Step 6: To find the point $x = 3$ again, press **F1** for trace, and move the cursor until the x -value is as close to $x = 3$ as possible. The y -value is the cost.

If you want a more accurate value, press **ZOOM** and repeat this procedure. When ZOOM is used, the range automatically changes. To return the range to the original values, press **ZOOM** and **F5** for ORG (original). This will redo the original graph and restore the original range.

To use TRACE on the TI-81, follow these steps:

Step 1: Press **TRACE** and use **▶** or **◀** to move the cursor.

Step 2: When you are at the point where the value of x is as close to $x = 3$ as possible, press **ZOOM**. A list of functions will appear on the screen.

Step 3: You can use **▲** and **▼** to move up and down the list. Press **ENTER** to select the function you want. BOX is the function you want.

Step 4: Since BOX is the first function, just press **ENTER** and move the cursor close to where the line crosses $x = 3$ and press **ENTER** again. This will set a starting point.

Step 5: Now move the cursor in any direction in order to go past the point. Notice that a line is drawn as you move the cursor. When you are sure you are past the point $x = 3$, move the cursor away from the line past the point again. This time you see a box drawn.

Step 6: Once the point is enclosed in the box, press **ENTER**. The section that was in the box will be drawn and expanded.

Step 7: To find the point $x = 3$, press **TRACE** and move the cursor until the x -value is as close to $x = 3$ as possible. The y -value is the cost.

For a more accurate value, press **ZOOM** and repeat this procedure. When **ZOOM** is used, the range automatically changes. Once you have found the value you want,

press **ZOOM** and move down to number

six (the standard function). Press **ENTER**

to reset the range to the standard values (-10 to $+10$).

According to the graphing calculator, the cost to fix the VCR is \$70.

Do the following questions using a graphing calculator or computer graphing program.

5. Graph the following equations. Set the range to -10 to $+10$.

- a. $y = 2x + 3$
- b. $y = 4x - 5$
- c. $y = -3x - 1$
- d. $y = -x + 2$

6. An autobody shop charges a flat fee of \$100 plus \$58 per hour. If it took 4.75 h to repair a car, how much did it cost? Round your answer to the nearest cent. Set the range to the values required for this problem.



For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.

Activity 3

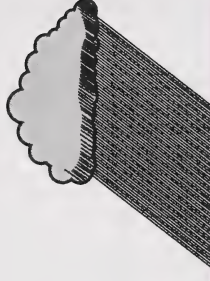
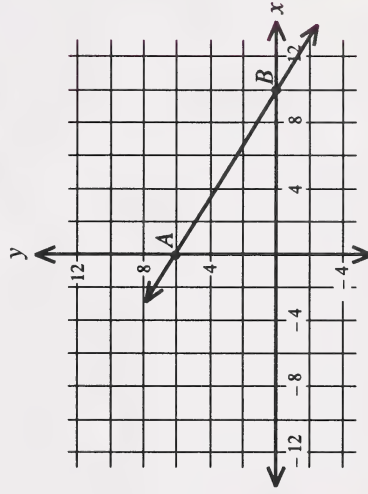


Determine x - and y -intercepts.

You can find x - and y -intercepts from a graph.

When you graph an oblique line, the line will cross the x - and y -axis. The point at which the line crosses an axis is called an intercept.

Look at the following graph. The graph crosses the x -axis at 10. The x -coordinate of point B is 10. This is the point where the line intercepts the x -axis and you call this coordinate the x -intercept. Similarly, the y -coordinate of point A is called the y -intercept. The graph crosses the y -axis at 6. The ordered pairs for points B and A are $(10, 0)$ and $(0, 6)$ respectively.



An oblique line is a line that is neither parallel or perpendicular to the axes.

A vertical line is parallel to the y -axis, and a horizontal line is parallel to the x -axis.

Now go to an example.

Example 8

A salesperson is paid a fixed salary of \$120 per day plus \$15 per hour for overtime. Graph this relation.

Solution:

Write an equation using x and y . x represents the number of hours of overtime, and y represents the daily income.

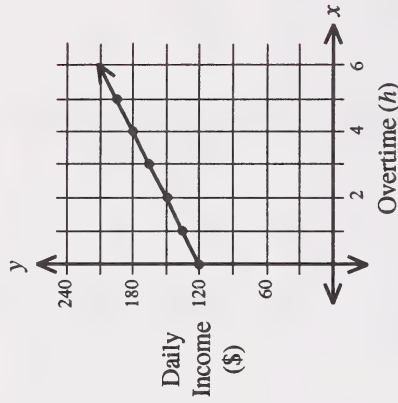
$$y = 120 + 15x$$

Construct a table of values to help you graph the relation.

x	y
1	135
2	150
3	165
4	180
5	195

The ordered pairs are (1, 135), (2, 150), (3, 165), (4, 180), and (5, 195).

Now plot the ordered pairs.



The y -intercept of this graph is 120. This is the straight pay for the day, without overtime pay.

The graph does not have an x -intercept because the salesperson cannot work negative hours overtime.

All the points are joined because the salesperson can work part of an hour and get paid.

Each ordered pair satisfies the equation $y = 120 + 15x$.

Now that you know how to find the x - and y -intercept from a graph, look at how you can find them from an equation.

When finding the x - and y -intercepts from an equation, notice that for any point on the x -axis, the y -coordinate is 0. Likewise for any point on the y -axis, the x -coordinate is 0.

Use your algebraic skills to find the x -intercept and the y -intercept when the equation of a line is $2x + 3y - 6 = 0$.

To find the x -intercept, let $y = 0$.

$$2x + 3y - 6 = 0$$

$$2x + 3(0) - 6 = 0$$

$$2x = 6$$

$$x = 3$$

The x -intercept is 3.

To find the y -intercept, let $x = 0$.

$$2x + 3y - 6 = 0$$

$$2(0) + 3y - 6 = 0$$

$$3y = 6$$

$$y = 2$$

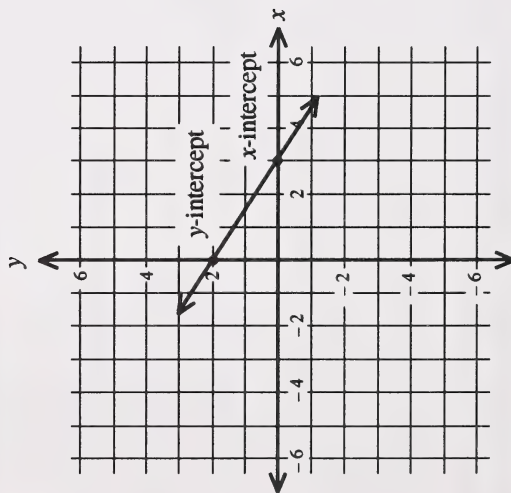
The y -intercept is 2.

You can write the x -intercept and the y -intercept as ordered pairs. For example, $(3, 0)$ and $(0, 2)$.

The intercepts provide you with an alternative method for graphing. When it comes to graphing the equation of a line, you can make a table of values and then draw the graph. Since the intercepts can be interpreted as points, you can use them to draw the graph. Using the intercepts, provide a quick method to graph an equation of a line.

Now graph $2x + 3y - 6 = 0$. Use the intercepts to make a table of values. The y -intercept is 2 and the x -intercept is 3.

x	y
0	2
3	0

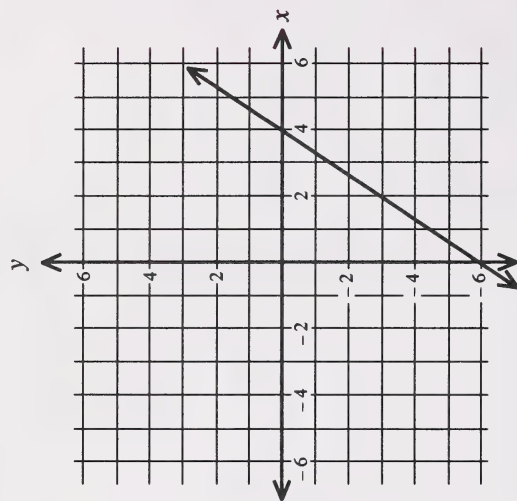


Now try the following questions.

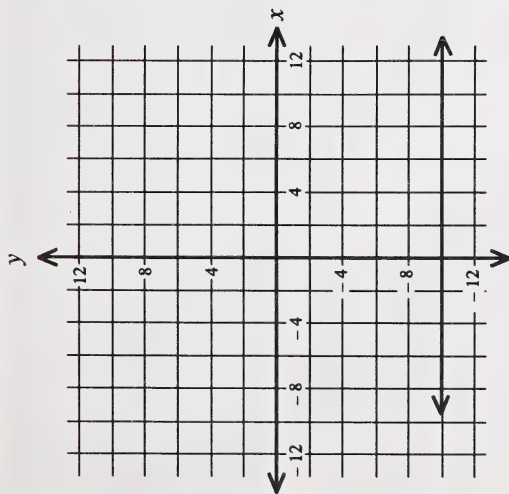
Do two parts from questions 1 and 2 and do questions 3 and 4. Do the others if you need more practice. Graph paper is provided in **Appendix B**.

1. State the x -intercept and y -intercept for each graph.

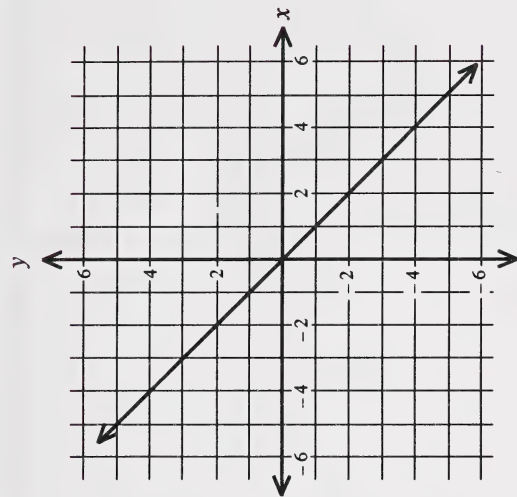
a.



b.



c.

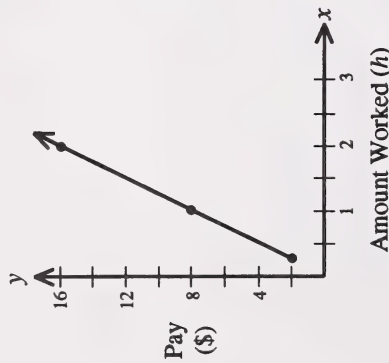


2. a. Find the x - and y -intercepts for $x - 2y = 8$. Graph the equation.

b. Find the x - and y -intercepts for $-3x + 2y - 12 = 0$. Graph the equation.

c. Find the x - and y -intercepts for $y = \frac{2}{3}x - 2$. Graph the equation.

3. There are two lines represented by equations given as $x = -3$ and $y = 5$ respectively. Give the intercepts for each line. What are the coordinates of the point of intersection of the two graphs?
4. The following graph shows the amount of pay per hour a farmhand receives. Use the graph to answer the following questions.



- a. What is the y-intercept?
- b. Why does the graph start at $(\frac{1}{4}, 2)$?
- c. Does the farmhand get paid if he puts in 2 h 10 min of work? Explain.



For solutions to **Activity 3**, turn to the **Appendix, Topic 4**.



A graphing calculator may be used to find the x - and y -intercepts. The equations must be in $y = mx + b$ form. Enter the equations; then, use the **TRACE** and **ZOOM** functions to find the intercepts. The more you use the **ZOOM** function, the more accurate your value will be.

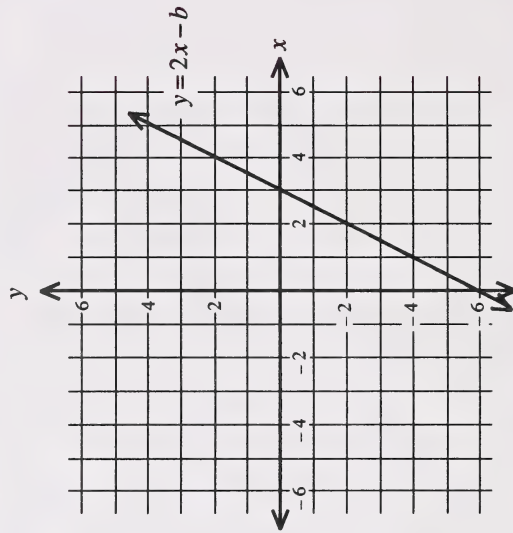
Example 9

Find the x - and y -intercepts for the following equations.

- $y = 2x - 6$

Solution:

Use a range of -10 to $+10$, and enter the equation.



Press **TRACE**, and move the cursor to the y-axis to find the x-intercept. Use ZOOM to make the y-value zero or as close to zero as possible. With integer intercepts, you can guess which integer-value the answer should be.

For example, you can guess that the x-intercepts in this question is 3 and the y-intercept is -6 just by using the TRACE function.

$$\bullet 3x + 2y = 5$$

Solution:

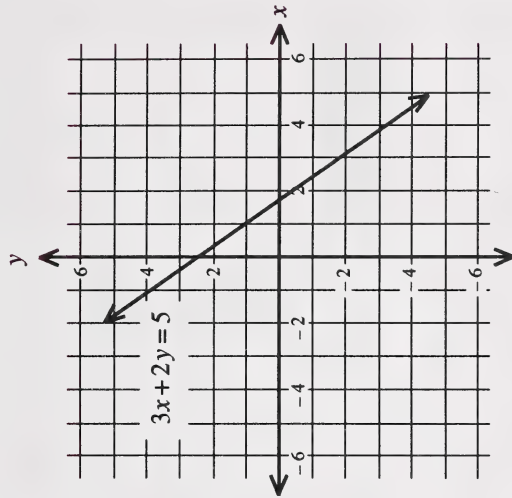
To find the x- and y-intercepts, you must put the equation into $y = mx + b$ form.

$$3x + 2y = 5$$

$$2y = -3x + 5$$

$$y = \left(-\frac{3}{2}\right)x + \frac{5}{2}$$

Now you can enter this equation into your calculator. Make sure you use brackets.



The x-intercept is 1.67 (rounded to two decimal places) or $1\frac{2}{3}$, and the y-intercept is 2.5.

Answer the questions that follow.

Find the x- and y-intercepts for the following equations.

5. $y = 3x - 4$

6. $y = -x + 5$

7. $y = -2x - 3$

8. $2x - 3y = 6$



For solutions to Activity 3, turn to Appendix A, Topic 4.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

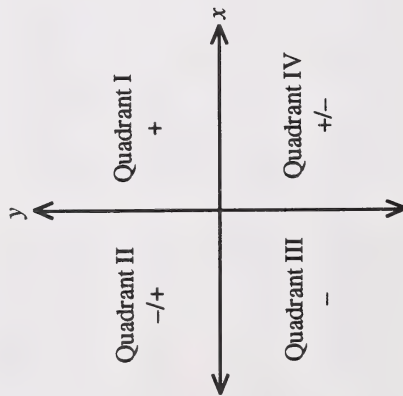
A coordinate plane is divided into four regions.

In Quadrant I, the x - and y -values are positive.

In Quadrant II, the x -value is negative and the y -value is positive.

In Quadrant III, the x - and y -values are negative.

In Quadrant IV, the x -value is positive and the y -value is negative.



When graphing a linear equation, a simple method to use is a table of values. Choose at least three values for x . Two values are sufficient, but the third value serves as a check.

A point has two coordinates: the x -coordinate and the y -coordinate. The x -coordinate is the x -value of the point and the y -coordinate is the y -value of the point. The point at which the graph crosses the y -axis is called the y -intercept. The point at which the graph crosses the x -axis is called the x -intercept.

A graph can be constructed using the x - and y -intercepts.



Do questions 1 to 3 or questions 4 to 8. Graph paper is provided in **Appendix B**.

1. In which quadrant are each of the following points located?

- a. $(-5, -2)$ b. $(7, -6)$
c. $(7, 1)$ d. $(-5, 4)$

2. Find the x - and y -intercepts for each of the following.

- a. $2x - 3y = 12$ b. $5x - y = 0$
c. $-2x + 4y = 8$ d. $2x = 18$

3. Use a table of values to graph the following.

- a. $x - 4y = 10$
b. $3x + 2y = -6$

4. In which quadrant is the point $(-4, -6)$? Why?

5. What is the y -coordinate in the ordered pair $(-12, 15)$?

6. In constructing a table of values for $y = 2x + 7$, usually you choose a set of values for x and then calculate the value of y . Why do you make this choice?

7. Find the x - and y -intercepts for each.

- a. $5x - 2y = 7$
b. $4x + 16y = 0$
c. $-y = 3$

8. Use the x - and y -intercepts to graph the following.

- a. $2x - 3y = 6$
b. $-y + 4x = -8$
c. $2x - 6 = 0$



For solutions to **Extra Help**, turn to **Appendix A, Topic 4**.



Extensions

Do either question 1 or question 2. Graph paper is provided in **Appendix B**.

1. The following table of values represents $y = mx$.

x	y
-4	-10
0	
1	
4	20
5	

- Find the value of m from the table.
- Complete the table.
- Write an equation showing the relationship between x and y .
- Use the table of values to graph the relation.
- Describe the graph of the relation between x and y .

2. For a skate-a-thon, Jim had pledged of \$0.50 per lap.
- If Jim covered 50 laps, how much would he collect?
 - How many laps did he complete if he collected \$32?
 - Write an equation relating the amount collected to the number of laps completed.
 - Draw the graph of the relation.
 - Describe the graph of this relation.
 - Should the graph go below the origin? Why or why not?



For solutions to **Extensions**, turn to **Appendix A, Topic 4**.

Topic 5 Slope and y-intercept



Introduction

In Topic 4, you learned that an equation like $3x - y + 2 = 0$ is a straight line when graphed. All the ordered pairs correspond to points that fall in a straight line. Such points are called collinear points. You will see that the previous equation can be written in various forms.

The following shows the previous equation written in slope y-intercept form.

$$\begin{aligned}3x - y + 2 &= 0 \\3x - y &= -2 \\-y &= -3x - 2 \\y &= 3x + 2\end{aligned}$$

In this topic you will look at some various forms and find out why there is a preferred form.



What Lies Ahead

Throughout the topic you will learn to

1. change the subject in formulas
2. write linear equations in $y = mx + b$ form, and identify the significance of m and b
3. determine the slope and the y-intercept of a line from its equation, and draw the graph
4. graph linear equations using slope and another point on the line
5. use a graphing calculator to study the effects of changing the values of m and b
6. describe and graph equations of lines parallel to the x-axis and lines parallel to the y-axis

Now that you know what to expect, turn the page to begin your study of slope and y-intercept.



Exploring Topic 5

Activity 1



Change the subject in formulas.

You will recall from an earlier course in math that relationships or formulas involving more than one variable can be rewritten in different forms. These rearrangements follow the guidelines that were developed in the unit on solving equations.

In the following example you will learn to isolate a particular variable.

Example 1

The circumference (C) of a circle is given by the formula $C = 2\pi r$, where r is the radius. Rewrite the formula in terms of r .

Solution:

$$C = 2\pi r$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{(Divide both sides by } 2\pi \text{ to isolate the } r.)$$

$$\frac{C}{2\pi} = r \quad \text{or} \quad r = \frac{C}{2\pi}$$

Usually equations of lines are written in $Ax + By + C = 0$ form. This is called the **general form** of an equation. However, it is sometimes advantageous to write an equation in a form where y is isolated. In other words, make y the subject. You will learn later why isolating y is important.

Example 2

Using $2x + y - 6 = 0$, solve for y .

Solution:

Transpose the $2x$ and the -6 . Remember to change the signs.

$$2x + y - 6 = 0$$

$$y = -2x + 6$$

In $Ax + By + C = 0$, A , B , and C are numerical values. In an equation such as $5x - 4y - 7 = 0$, $A = 5$, $B = -4$, and $C = -7$. In $-3x + y = 0$, $A = -3$, $B = 1$, and $C = 0$.

To isolate y means to have only y on one side of the equation.

Transpose means to move a term from one side of an equation to the other. Change the sign of the term when you do this.

Example 3

Solve for y in $8x + 2y + 12 = 0$.

Solution:

$$8x + 2y + 12 = 0$$

(Transpose.)

$$2y = -8x - 12$$

$$\frac{2y}{2} = \frac{-8x}{2} - \frac{12}{2}$$

(Divide each term by 2.)

$$y = -4x - 6$$

Example 4

Solve for x in $2x - 3y = 4$.

Solution:

$$2x - 3y = 4$$

$$2x = 3y + 4$$

$$x = \frac{3}{2}y + \frac{4}{2}$$

$$x = \frac{3}{2}y + 2$$

You can see from the previous example that you can isolate any variable.

Try the following exercises.

Complete at least 1b, 1c, 2b, and 3.

1. Solve for y in each equation.

a. $4x - 3y = 6$

b. $-x + \frac{1}{2}y - 2 = 0$

c. $3xy - 12 = 0$

2. Make x the subject in each equation.

a. $2x + y = -2$

b. $\frac{3}{4}x - 2y = 24$

c. $7y - 4x - 28 = 0$

3. The area of a triangle is $A = \frac{1}{2}bh$. Write h in terms of A and b .

4. If $I = Prt$, isolate r .



For solutions to **Activity 1**, turn to **Appendix A**, **Topic 5**.

Activity 2



Write linear equations in $y = mx + b$ form, and identify the significance of m and b .

Now you will see the benefits of changing equations to a new form. The equations you will be concerned with for now contain the common variables x and y .

Study the following example.

Example 5

Graph the equation $y = \frac{3}{2}x - 3$.

Solution:

You have learned to graph a linear relation using the intercepts. Find the x - and y -intercept for $y = \frac{3}{2}x - 3$.

$$\text{When } y = 0, 0 = \frac{3}{2}x - 3$$

$$-\frac{3}{2}x = -3$$

$$-\frac{3}{2}x \times \left(-\frac{2}{3}\right) = -\frac{3}{1} \times \left(-\frac{2}{3}\right) \quad \left(\begin{array}{l} \text{Multiply} \\ \text{by } -\frac{2}{3}. \end{array}\right)$$
$$x = 2$$

The x -intercept is 2. Thus, the ordered pair is $(2, 0)$.

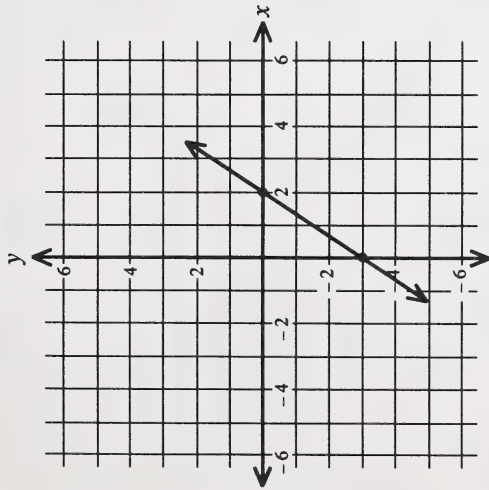
$$\text{When } x = 0, y = \frac{3}{2}(0) - 3$$

$$y = -3$$

The y -intercept is -3 . Thus, the ordered pair is $(0, -3)$.

The general form for this equation is $3x - 2y - 6 = 0$.

The x - and y -intercepts will help you graph the equation.



You can find the slope of this graph from the ordered pairs using the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 0}{0 - 2} \\ &= \frac{-3}{-2} \\ &= \frac{3}{2} \end{aligned}$$

Look closely at the coefficient of x in the equation and compare it with the value of m . They are both equal to $\frac{3}{2}$. Also, examine the y -intercept of the graph, the constant term of the equation, and the value of the y -intercept. In each case the value is -3 .

In $y = \frac{3}{2}x - 3$, the slope is the numerical coefficient of x , and the constant term is the y -intercept.

You have developed a useful form of the equation. This form is $y = mx + b$. Now by inspection, you can find the slope and y -intercept.

In $y = mx + b$, m and b are parameters. These parameters give a straight line, represented by an equation, its unique characteristics.

You can compare graphs or equations with respect to direction, steepness, and position of the y -intercept using parameters. An equation expressed in $y = mx + b$ form is said to be in slope y -intercept form.



A linear equation is defined by $y = mx + b$, where m is the slope of the line and b is the y -intercept.

Recall: When the slope is positive, the graph rises to the right. When the slope is negative, the graph falls to the right.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

Note: Slope y -intercept form is as follows:

$$\begin{array}{c} \text{slope} \\ \downarrow \\ y = mx + b \\ \uparrow \\ \text{y-intercept} \end{array}$$

A parameter represents a constant or a variable in a mathematical expression.

In the next example you will see how to express an equation in slope y -intercept form.

Example 6

Find the slope and the y -intercept of $3x - 2y = 12$, $x \in R$.

Solution:

Write the equation in slope y -intercept form.

$$\begin{aligned} 3x - 2y &= 12 \\ -2y &= -3x + 12 \\ \frac{-2}{-2}y &= \frac{-3}{-2}x + \frac{12}{-2} \\ y &= \frac{3}{2}x - 6 \end{aligned}$$

The slope is $\frac{3}{2}$ and the y -intercept is -6 .

The slope y -intercept form helps to solve problems by making comparisons. The following example will demonstrate this.

Example 7

The lines $5x + 2y = -4$ and $-x - 3y = k$ have the same y -intercepts. Find the value of k .

Solution:

Write each equation in slope y -intercept form.

$$\begin{aligned} 5x + 2y &= -4 \\ 2y &= -5x - 4 \\ y &= \frac{-5}{2}x - 2 \\ -x - 3y &= k \\ -3y &= x + k \\ y &= \frac{x}{-3} + \frac{k}{-3} \end{aligned}$$

Since the y -intercepts are equal,

$$\begin{aligned} \frac{k}{-3} &= -2 \\ (-3)\left(\frac{k}{-3}\right) &= -3(-2) \\ k &= 6 \end{aligned}$$

If a point lies on a line, its coordinates satisfy the equation of the line. Verify this in the next example.

$x \in R$ means that x is a real number.

Example 8

The line $y = mx - 5$ contains the point $(6, -1)$. Find the value of m .

Solution:

Since the coordinates are on the line, substitute $(6, -1)$ for x and y into the equation of the line.

$$-1 = m(6) - 5$$

$$-1 = 6m - 5$$

$$-6m = -5 + 1$$

$$-6m = -4$$

$$\frac{-6}{-6}m = \frac{-4}{-6}$$

$$m = \frac{2}{3}$$

The slope is $\frac{2}{3}$.

Go to the following exercise and apply what you have learned about the $y = mx + b$ form.

Do questions 1c, 2c, 2d, 3a, 3b, 4, and 5. If you are still having difficulty understanding your work, do the other questions. Graph paper is provided in **Appendix B**.

1. Give the slope and the y -intercept for each of the following.

a. $y = 3x + 7$

b. $y = -\frac{1}{2}x - 4$

c. $y = \frac{4}{5}x - 2$

2. Write the following equations in slope y -intercept form.

a. $x - 2y = 6$

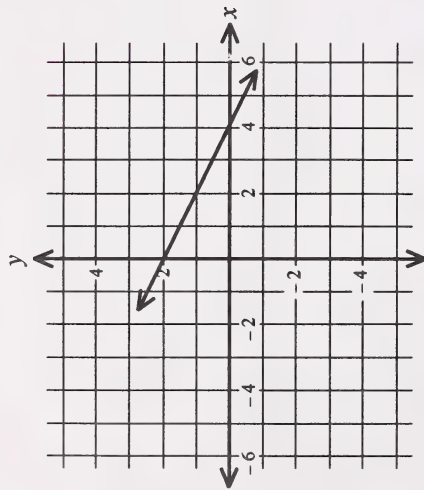
b. $2x + 5y + 15 = 0$

c. $-3x - 4y + 12 = 0$

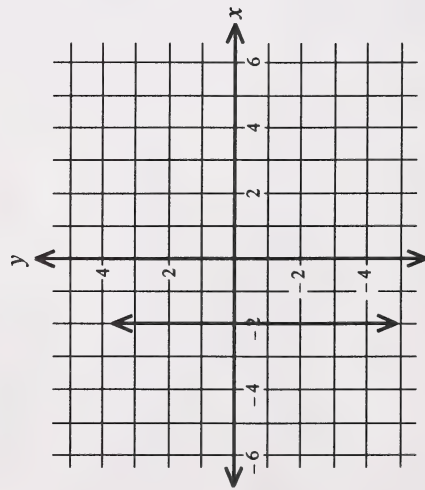
d. $2x - \frac{1}{2}y = 10$

3. Find the slope and the y-intercept of each line.

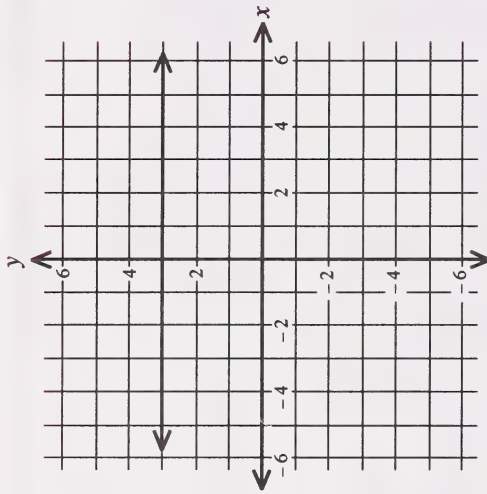
a.



b.



c.



4. With the help of graphs, compare the slopes of $y = 2x - 1$ and $y = \frac{1}{2}x - 1$. What conclusion can you draw from this comparison?

5. a. Use a table of values to draw the graph for $y = \frac{2}{3}x$ and $y = \frac{2}{3}x + 2$.

- b. Use the graphs in question 5a to compare the equations.



For solutions to **Activity 2**, turn to **Appendix A**, **Topic 5**.

Activity 3



Determine the slope and the y-intercept of a line from its equation, and draw the graph.

Using intercepts is the simplest method to graph equations. However, if one or both intercepts are fractions, the graph may be difficult to plot accurately.

A second method of graphing a linear equation is to construct a table of values. You learned this method in **Topic 4** of this unit.

Another method for graphing a linear equation is based on two numbers: the slope and the y-intercept. The equation must be in slope y-intercept form for convenience of graphing. Recall this form from the definition where $y = mx + b$. In this equation, m is the coefficient of x and represents the slope, and b is the constant value and represents the y-intercept.

Example 9

Graph the following equations.

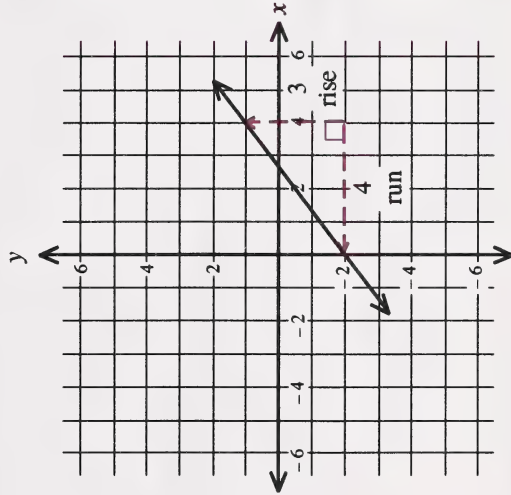
- $y = \frac{3}{4}x - 2$

Solution:

The slope is $\frac{3}{4}$ and the y-intercept is -2 .

$$m = \frac{\text{rise}}{\text{run}} = \frac{3}{4}$$

Locate -2 on the y-axis (y-intercept). From the point $(0, -2)$, move four units to the right and three units up. The point where you stop is a point on the graph. In this example, it is $(4, 1)$. Now draw the line through $(4, 1)$ and $(0, -2)$.



You can find other points on this line by continuing this way or by moving four units to the left and three units down.

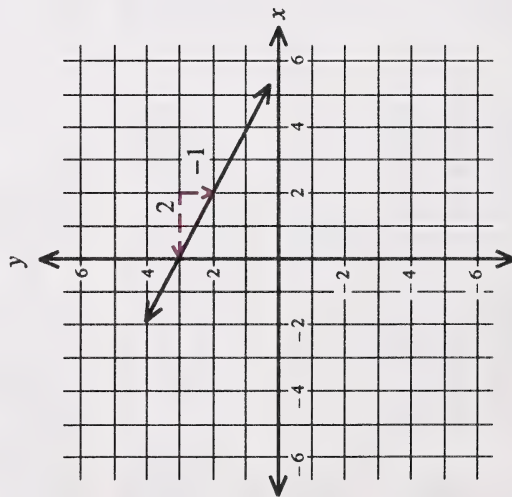
Notice that the graph rises to the right. Since the slope is positive, the movement is in a positive direction from $(0, -2)$ to the right and up.

- $y = -\frac{1}{2}x + 3$

Solution:

The slope is $-\frac{1}{2}$ and the y -intercept is 3.

From the point $(0, 3)$, move two units to the right and one unit down. The point $(2, 2)$ is on the line. Draw the line through $(0, 3)$ and $(2, 2)$.

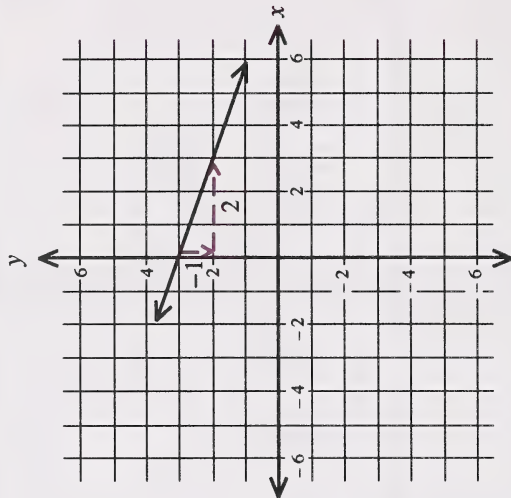


Here the slope is negative; therefore, the graph falls to the right.

Since the slope is negative, rewrite $m = -\frac{1}{2}$ as $m = \frac{-1}{2}$. This will help you to move in the proper direction to find a point for the line.

Notice that a move from $(0, 3)$ is made in a positive direction (right) and then in a negative direction (down), since the run is 2 and the rise is -1 .

Starting from $(0, 3)$, you can also go down one and to the right three.



Example 10

If the graph of $y = \frac{2}{5}x + b$ passes through the point $(-5, 4)$, find the y -intercept.

Solution:

Since the line passes through $(-5, 4)$, it will satisfy the equation $y = \frac{2}{5}x + b$.

$$y = \frac{2}{5}x + b$$

$$4 = \frac{2}{5}(-5) + b$$

$$4 = -2 + b$$

$$6 = b$$

The y -intercept is 6.

Thus, the equation is $y = \frac{2}{5}x + 6$.

Example 11

The point $(1, 2)$ satisfies the equation $y = mx + 4$. Find the slope m , and draw the graph.

Solution:

Substitute $(1, 2)$ in $y = mx + 4$.

$$y = mx + 4$$

$$2 = m(1) + 4$$

$$2 - 4 = m$$

$$-2 = m$$

The slope is -2 .

Thus, the equation is $y = -2x + 4$.

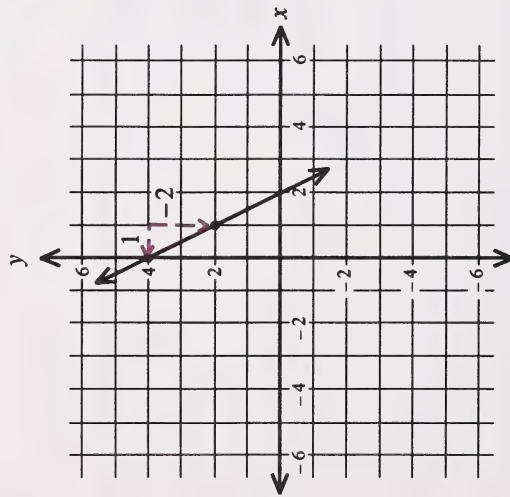
Graph the equation.

Since the y -intercept is 4, $(0, 4)$ is a point on the graph.

Since the slope is negative, the line will fall to the right.

$$\begin{aligned} m &= -2 \\ &= \frac{-2}{1} \end{aligned}$$

From the point $(0, 4)$, move one unit to the right and two units down. The point $(1, 2)$ is on the line. Draw a line through $(0, 4)$ and $(1, 2)$. Note that since the point $(1, 2)$ was given as a point on the line it is not necessary to use the slope. The points $(0, 4)$ and $(1, 2)$ are enough to draw the graph.



For more information on finding slope given the equation, you may go to the computer program titled *Algebra Drill and Practice*,¹ Part 3, Section 2. Follow some examples first; then do as many questions as you find necessary.



¹ *Algebra Drill and Practice* is a title of *Conduit*.

Now try the following questions.

Complete questions 1a, 1d, 1e, 2c, 2d, 3a, and 3c. If you need more practice, do the other questions. Graph paper is provided in **Appendix B**.

1. Identify the slope and the y -intercept for each line.

a. $y = \frac{7}{8}x - 5$

b. $y = -2x + 2$

c. $y = 4x$

d. $y = -\frac{2}{3}x - 3$

e. $y = \frac{-x}{7}$

2. Find the slope for each equation if its graph passes through the given point; then rewrite the equation.

a. $y = mx - 5$ $(-3, -8)$

b. $y = mx + 2$ $\left(-2, \frac{1}{2}\right)$

c. $y = mx - 7$ $(7, -2)$

d. $y = mx + 1$ $(-4, 2.5)$

3. If each point satisfies the given equation, find the y-intercept; then, graph the equation.

a. $y = -\frac{1}{2}x + b$ $(-2, 4)$

b. $y = \frac{2}{5}x + b$ $(5, 3)$

c. $y = \frac{-3}{2}x + b$ $(-4, 4)$



For solutions to **Activity 3**, turn to **Appendix A, Topic 5**.



Activity 4



Graph linear equations using slope and another point on the line.

A line can be drawn through a given point with a given slope. You will see how this works in the following examples.

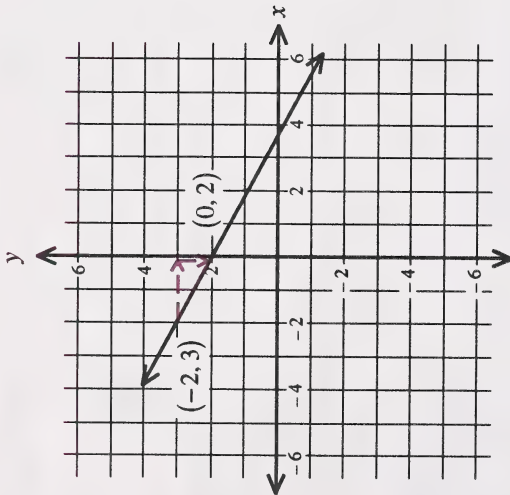
Example 12

- Draw a line with a slope of $-\frac{1}{2}$ through the point $(-2, 3)$.

Solution:

Begin at $(-2, 3)$ and move two units to the right and one unit down. This yields the point $(0, 2)$. Draw a line through $(-2, 3)$ and $(0, 2)$.

You can verify that your graph is correct by using a table of values and plotting the ordered pairs.



- Write the equation of the line.

Solution:

The slope y-intercept form is $y = mx + b$.

$$m = -\frac{1}{2}$$

From the graph, the y-intercept is 2.

Therefore, $b = 2$.

Thus, the equation of the line is $y = -\frac{1}{2}x + 2$.

Example 13

- Graph the equation $y = \frac{2}{3}x + b$ if it passes through $(-1\frac{1}{2}, 4)$.

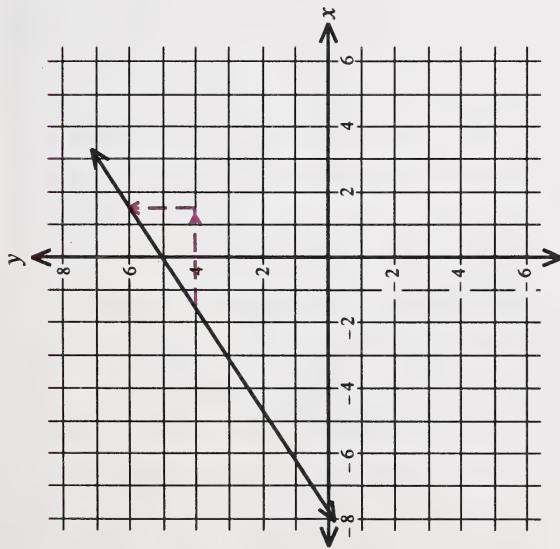
Solution:

To draw the graph of $y = \frac{2}{3}x + b$, you need to find the slope and a point which the line passes through.

The slope is $\frac{2}{3}$ and the point is $(-1\frac{1}{2}, 4)$.

Starting at $(-1\frac{1}{2}, 4)$, move three units to the right and two units up. The new point is $(1\frac{1}{2}, 6)$.

Draw a line through $(-1\frac{1}{2}, 4)$ and $(1\frac{1}{2}, 6)$.



- Calculate the value of b .

Solution:

Since $(-1\frac{1}{2}, 4)$ are the coordinates of a point on the line $y = \frac{2}{3}x + b$, you can find b by substitution.

Substitute $(-1\frac{1}{2}, 4)$ in $y = \frac{2}{3}x + b$.

$$y = \frac{2}{3}x + b$$

$$4 = \frac{2}{3}\left(-1\frac{1}{2}\right) + b$$

$$4 = \frac{2}{3}\left(-\frac{3}{2}\right) + b$$

$$4 = -1 + b$$

$$b = 5$$

- From the graph, find the value of b .

Solution:

From the graph, the y -intercept is 5.

The y -intercept is 5. This is the same value found from the graph.

Try the following questions.

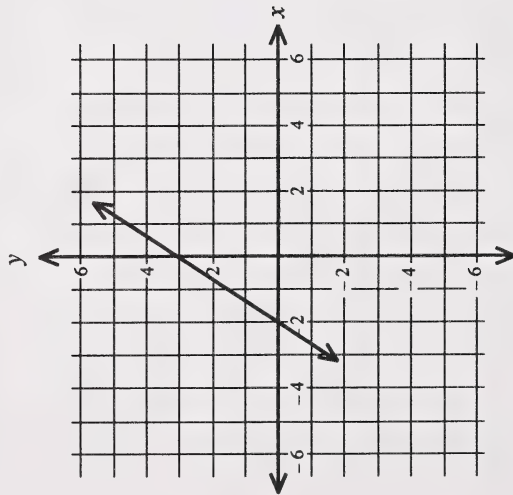
Do questions 1a, 1c, 2, 3a, 3c, and 4. Do the other questions for more practice. Graph paper is provided in **Appendix B**.

1. Draw the graph of each line if a point and its slope are given.

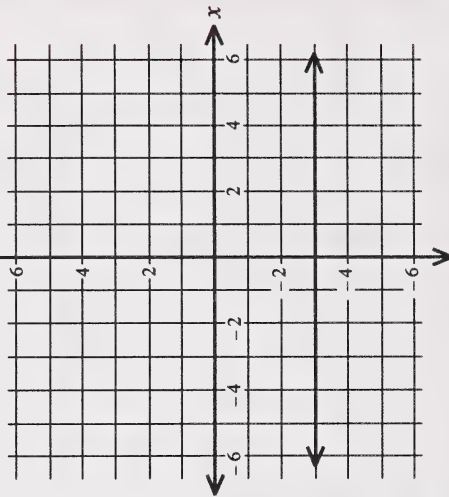
- a. $(3, 1)$, $m = -2$
- b. $(-2, -5)$, $m = \frac{3}{4}$
- c. $(-3, 2)$, $m = -\frac{2}{5}$
- d. $(3, 0)$, $m = -\frac{3}{2}$

2. Write the equation for each line given.

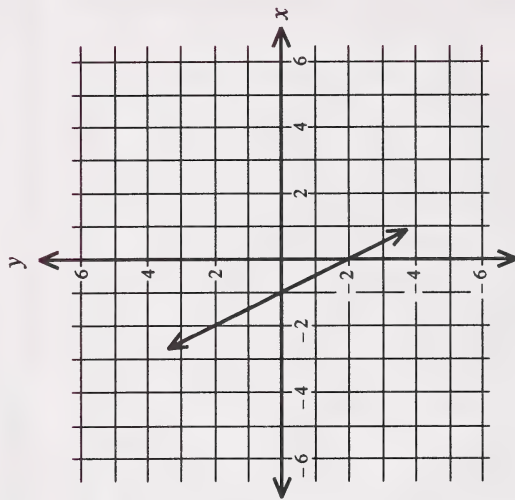
a.



c.



b.

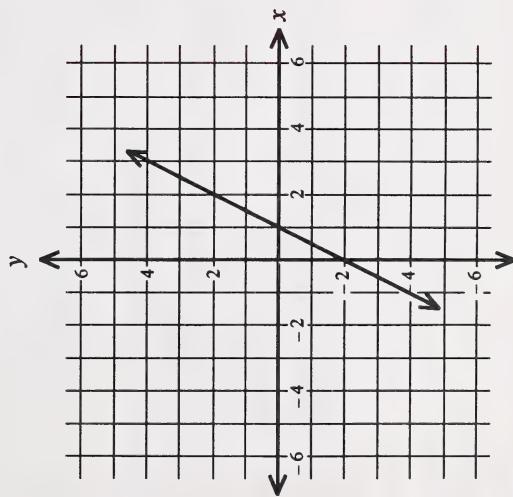


3. Graph each line using the slope and a given point. State the y -intercept and the x -intercept by reading them off the graphs. Calculate the y -intercept using the given point.

a. $y = 2x + b$ $(-2, -2)$ b. $y = -\frac{3}{2}x + b$ $(2, 3)$

c. $y = \frac{3}{4}x + b$ $\left(-2, 1\frac{1}{2}\right)$

4. Explain how to obtain the slope of the line on the following graph.



For solutions to **Activity 4**, turn to **Appendix A, Topic 5**.

The slope of a line can be calculated by using the ratio $\frac{\text{rise}}{\text{run}}$.

Activity 5



Use a graphing calculator to study the effects of changing the values of m and b .

A graphing calculator or computer graphing program is especially useful in the study of the effects of various values of m and b in linear equations.



Use a graphing calculator for the following questions on the effects of changing the values of m and b .

1. Enter $y = x$, $y = 2x$, and $y = 3x$ on your graphing calculator.
 - a. What happens to the slope as m changes from 1 to 2 to 3?
 - b. What will happen to the line when $y = 10x$? Use your graphing calculator to check your answer.
 - c. What can you conclude about the effect on the slope when the positive value of m is increasing?
2. Enter $y = -x$, $y = -2x$, and $y = -3x$ on your graphing calculator.
 - a. What happens to the slope as m changes from -1 to -2 to -3 ?

- b. What will happen to the line when $y = -10x$? Use your graphing calculator to check your answer.
 - c. What can you conclude about the effect on the slope when the negative value of m is increasing?
3. How do the lines of the equations with positive values of m compare to the lines of the equations with the negative values of m ?
4. Enter $y = x$, $y = \left(\frac{1}{2}\right)x$, and $y = \left(\frac{1}{3}\right)x$ on your graphing calculator.
 - a. What happens to the slope as m changes from 1 to $\frac{1}{2}$ to $\frac{1}{3}$?
 - b. What will happen to the line when $y = \left(\frac{1}{10}\right)x$? Use your graphing calculator to check your answer.
 - c. What can you conclude about the effect on the slope when the value of m is decreasing?
 - d. What will happen to the line when $y = \left(-\frac{1}{2}\right)x$? Use your graphing calculator to check your answer.
5. Enter $y = x$, $y = x + 1$, and $y = x + 2$ on your graphing calculator.
 - a. What happens to the line as b changes from 0 to 1 to 2?
 - b. What will happen to the line when $b = 10$?

6. Keep the graphs from the previous question and enter $y = x - 1$ and $y = x - 2$.

- What happens to the line as b changes from 0 to -1 to -2 ?
- How do these lines relate to the lines in question 5?
- What will happen to the line when $b = -10$?

7. Study the effects of changing b with $y = -x$ on your own. Compare the effects of changing the value of b with $y = x$ to $y = -x$.



For solutions to **Activity 5**, turn to **Appendix A, Topic 5**.

Activity 6



Describe and graph equations of lines parallel to the x -axis and lines parallel to the y -axis.

Many of the concepts that have been discussed in this topic relate to oblique (slanted) lines. Lines that are horizontal or vertical often require somewhat different approaches.

In this activity, you are going to investigate the properties of horizontal and vertical lines.

When you are faced with graphing an equation, use one of the following methods:

- table of values
- both intercepts (intercept form)
- slope and one of the intercepts (slope y -intercept form)

These methods will work for any equation that includes an x - and y -term.

However, there are some equations in which the x - or y -term may be missing. In such cases, you may have some difficulty using the previous methods. What you need here is a different plan.

Study the following example to see how to handle cases in which you are missing one of the variables.

Example 14

Graph the line $2y + 6 = 0$.

Solution:

The equation can be solved using one of the two methods. First, solve for y .

$$2y = -6$$

$$y = -3$$

In slope y -intercept form, the equation would look as follows:

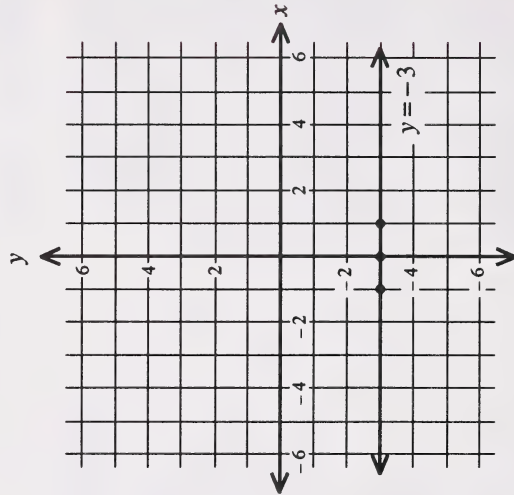
$$y = 0x - 3$$

You can see that the y -intercept for this equation is -3 , and the slope is 0 . The graph of this line would be a horizontal line, three units below the x -axis.

This line can also be graphed using a table of values.

x	y
-1	-3
0	-3
1	-3

No matter which number you substitute for x , you will get -3 for y .



Any equation that can be written in the form $y = k$ is a horizontal line, and it passes through the value of k on the y -axis.

Remember: The slope of a horizontal line is 0 .

Example 15

Graph the line $3x - 7 = 2$.

Solution:

The equation $3x - 7 = 2$ cannot be written in y -intercept form because there is no y -term. Instead, solve the equation for x .

$$3x - 7 = 2$$

$$3x = 9$$

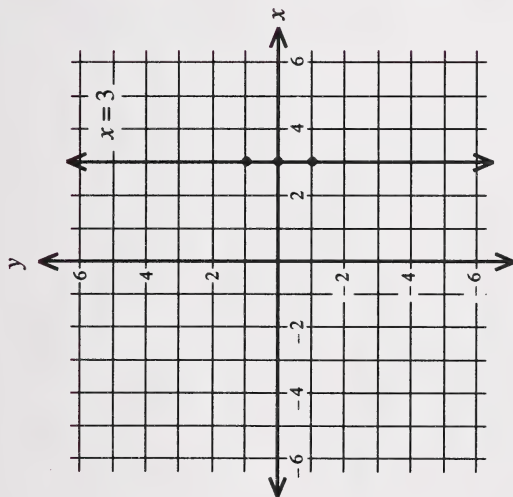
$$x = 3$$

Now you can rewrite this equation inserting a y -term. (Note that $0y$ is used so the value of the equation is not changed.)

$$x = 3 + 0y$$

Now make a table of values to graph this equation.

x	y
3	-1
3	0
3	1



Now try the following exercise.

Remember: The slope of a vertical line is undefined.

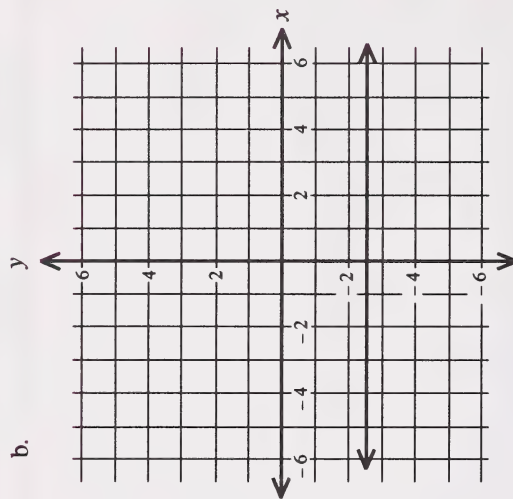
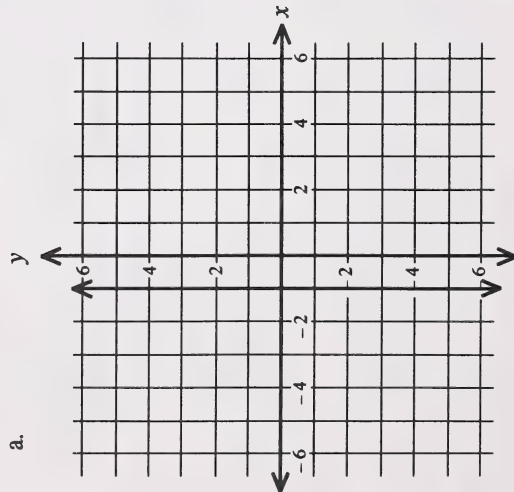
Any equation that can be written in the form $x = k$ is a vertical line, and it passes through the value of k on the x -axis.

Complete at least two parts of questions 1 and 3 and either question 2a or 2b. The others may be completed if you need more practice.

1. Draw the following on graph paper. Give the slope of each. Graph paper is provided in **Appendix B**.

- a. $x = 5$
- b. $2y = 8$
- c. $3y + 8 = -4$
- d. $2x + 6 = 3$

2. Write the equations for each of the following graphs.



3. Which of the following pairs of points determine horizontal line segments and which determine vertical line segments?

- a. $(0, 0)$, $(0, -4)$
- b. $(2, -2)$, $(-2, -2)$
- c. $(-1, 0)$, $(3, 0)$



For solutions to **Activity 6**, turn to **Appendix A**, **Topic 5**.



A horizontal line is parallel to the x -axis and a vertical line is parallel to the y -axis.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

When you isolate a variable, you are writing one variable in terms of the other variables. The coefficient of the isolated variable is positive one. Isolating a variable makes substitution easier and speeds up your calculation. It is particularly important for graphing. Notice that the form $y = mx + b$ is very useful in finding the slope and the y -intercept of the graph. This form also helps you to construct a table of values for graphing.

If you are having difficulty making headway in this topic, try these questions.

Do the odd- or even-numbered questions. Graph paper is provided in **Appendix B**.

1. Isolate x in the following.

a. $8xy = 36yz$

b. $2x + 7 = 4m - 3$

c. $\frac{3y}{x} = 2$

2. Make h the subject of each equation.

a. $A = 4bh$

b. $64T^2 = h^2$

c. $2\sqrt{h} = 6mn$

3. Write each equation in the form $y = mx + b$.

a. $x - y = 4$

b. $2x + y = 12$

c. $-2x + 3y = 15$

4. Make y the subject of each equation.

a. $4y = -12$

b. $5x - 2y = 10$

c. $4y + x = 4$

5. Give the slope and the y-intercept for each of the following.

a. $y = -5x + 2$

b. $y = \frac{3}{4}x - 8$

c. $y = -\frac{2}{3}x + 4$

6. Write each equation in slope y-intercept form. Give the slope and the y-intercept; then graph the equations.

a. $2y + 5x = 20$

b. $-2x + 2y = -5$



For solutions to **Extra Help**, turn to **Appendix A**, **Topic 5**.



Extensions

If you would like a more challenging exercise, do the following. Do any two parts of question 1 and either question 2 or 3. Graph paper is provided in **Appendix B**.

1. Solve for the indicated variable in terms of the other variables.

a. $d = \frac{1}{2}gt^2$; t

b. $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$; V_1

c. $E = \frac{WV^2}{2g}$; V

d. $c = \sqrt{2gh}$; g

2. Write the equation of a line in standard form that has a slope of $-\frac{5}{12}$ and passes through $(-2, 12)$.

3. For the equation $7x + 2y = -6$, do the following:

- Write an equivalent equation in slope y-intercept form.
- Determine the slope of the corresponding line.
- Determine the y-intercept.
- Draw the graph of the line using slope y-intercept form.



For solutions to **Extensions**, turn to **Appendix A**, **Topic 5**.

Topic 6 Equations with Unique Conditions



Introduction

This topic will link the various concepts covered earlier to the important skill of determining the equations of various linear graphs.

You will be working in reverse here using the slope, y -intercept, and specific points of a line to determine the equation of the line.



What Lies Ahead

Throughout the topic you will learn to

1. rewrite linear equations in $Ax + By + C = 0$ form
2. determine the equation of a line given the slope and a point on the line
3. determine the equation of a line passing through two given points
4. determine the equations of horizontal and vertical lines
5. determine the equation of a line given one point and the equation of a line parallel or perpendicular to it

Now that you know what to expect, turn the page to begin your study of equations with unique conditions.



Exploring Topic 6

Activity 1



Rewrite linear equations in $Ax + By + C = 0$ form.

In **Topic 5** of this unit, you learned to write the equation of a straight line in $y = mx + b$ form. This is the **slope y-intercept form**. From this form you can easily identify the slope and the y-intercept.

A linear equation can also be expressed in the form $Ax + By + C = 0$, where A , B , and C are constants. This is the general equation in standard form. For an equation to be in standard form means the x -term is first, followed by the y -term, and the constant on the left side of the equation. You can transform equations from one form to the other as you will see in the following example.

Example 1

Express the equation $y = 3x + 7$ in $Ax + By + C = 0$ form, and give the values of A , B , and C .

Solution:

You need an equation with all the terms on one side. Transfer all the terms to one side of the equation and equate it to zero. In $y = 3x + 7$, it is easier to transfer the y -term to the right side since you have only one term to move.

$$\begin{aligned}\therefore y - y &= 3x + 7 - y && \text{(Subtract } y \text{ from both sides.)} \\ 0 &= 3x + 7 - y\end{aligned}$$

Now write the equation in its correct form.

$$3x - y + 7 = 0$$

Therefore, $A = 3$, $B = -1$, and $C = 7$.

Any equation given in $y = mx + b$ form can be similarly changed to $Ax + By + C = 0$ form.

You have learned how to rewrite equations in $y = mx + b$ form (slope y-intercept form) into $Ax + By + C = 0$ form.

$$\begin{aligned}0 &= 3x + 7 - y && \text{can be written as} \\ 3x - y + 7 &= 0.\end{aligned}$$

Any term in an equation can be moved from one side to the other side. In the next example you will move two terms from the right side to the left side of the equation.

Example 2

Write the equation $2y = 5x - 2$ in standard form. Identify the values of A , B , and C .

Solution:

$$2y = 5x - 2$$

$$2y - 5x = 5x - 5x - 2 \quad (\text{Subtract } 5x \text{ from both sides.})$$

$$2y - 5x = -2$$

$$2y - 5x + 2 = -2 + 2 \quad (\text{Add } 2 \text{ to both sides.})$$

$$2y - 5x + 2 = 0$$

$$-5x + 2y + 2 = 0 \quad (\text{Write in correct order.})$$

Thus, $A = -5$, $B = 2$, and $C = 2$.

Note that $-5x + 2y + 2 = 0$ may be written with x having a positive coefficient.

Multiply $-5x + 2y + 2 = 0$ by -1 .

The equation now becomes $5x - 2y - 2 = 0$.

Thus, $A = 5$, $B = -2$, and $C = -2$.

Go to the **Extensions** section to see equations that have fractional coefficients.

Try the following exercise. Do questions 2, 3b, and 4. If you need more practice, complete the others.

1. Write $4y - 3x = 12$ in standard form. What are the values of A , B , and C ?

2. Write the equations $3x - 5y = -2$ and $9x = 15y - 6$ in $Ax + By + C = 0$ form. Show that these equations are equivalent.

3. Write the following equations in standard form. Identify the values of A , B , and C .

- a. $5y - 3 = -7x$

- b. $y = \frac{1}{2}x - 2$

- c. $8x = -5$

4. Find the x -intercept if $Ax + By + C = 0$.

(Hint: Let $y = 0$.)



For solutions to **Activity 1**, turn to **Appendix A, Topic 6**.

Remember: When you move a term from one side of the equation to the other side, change the sign of the term.

For example,

$$2y = 5x - 2$$

$$-5x + 2y = -2$$

$$-5x + 2y + 2 = 0$$

$$-1(-5x + 2y + 2) = -1(0)$$

$$5x - 2y - 2 = 0$$

Both $-5x + 2y + 2 = 0$ and $5x - 2y - 2 = 0$ are equivalent.

The one with the positive x -coefficient is more acceptable.

Activity 2



Determine the equation of a line given the slope and a point on the line.

Up to this point, you have been working with information relating to points, lines, and equations. You are now going to concentrate on finding the equation of lines given specific information. First you will look at a situation where the slope of a line and one point on the line are given.

A straight line may be determined in three ways:

- the slope and the y -intercept
- the x -intercept and the y -intercept
- a table of values

Can you determine the equation of a line if you know its slope and any one point on the line?

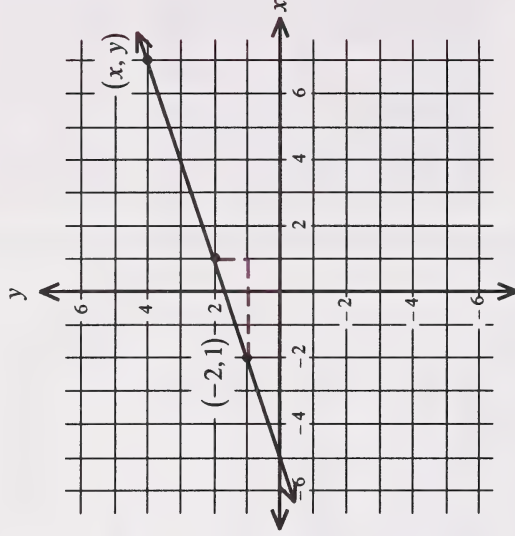
Look at the following example.

Example 3

Find the equation of a line with slope $\frac{1}{3}$ passing through $(-2, 1)$.

Solution:

Graph the line and let $P(x, y)$ be any point on the line.



Use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1) = (-2, 1)$ and (x_2, y_2) is any point (x, y) on the line.

Substitute $(-2, 1)$ into the formula.

$$\frac{1}{3} = \frac{y-1}{x-(-2)}$$

$$\frac{1}{3} = \frac{y-1}{x+2}$$

$$3(y-1) = 1(x+2) \quad (\text{Cross multiply.})$$

$$3y - 3 = x + 2$$

$$3y - 3 - x = x + 2 - x \quad (\text{Subtract } x \text{ from both sides.})$$

$$3y - 3 - x = 2 - x$$

$$3y - 3 - x - 2 = 2 - 2 \quad (\text{Subtract 2 from both sides.})$$

$$3y - 3 - x - 2 = 0$$

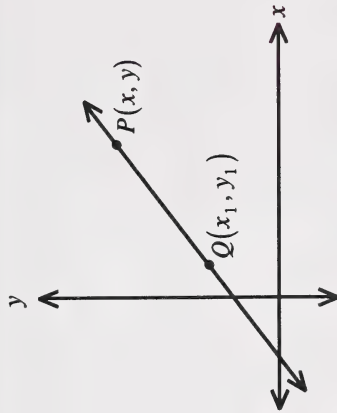
$$3y - x - 5 = 0 \quad (\text{Multiply both sides by } -1 \text{ and rearrange the terms.})$$

$$x - 3y + 5 = 0$$

The equation of the line is $x - 3y + 5 = 0$. The equation is in $Ax + By + C = 0$ form.

You will now generalize a procedure to find the equation of any line given its slope and one point on the line.

The coordinates of the given point are represented by $Q(x_1, y_1)$. Any other point on the line is $P(x, y)$. The slope of the line is m .



Let $x_2 = x$, $y_2 = y$. Substitute into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = y - y_1 \quad (\text{Cross multiply. Remember that } m \text{ is the same as } \frac{m}{1}.)$$



The required equation is $y - y_1 = m(x - x_1)$. This is called the **point-slope form** of the equation.

Now you can use the point-slope form to determine the equation of the line in the following example.

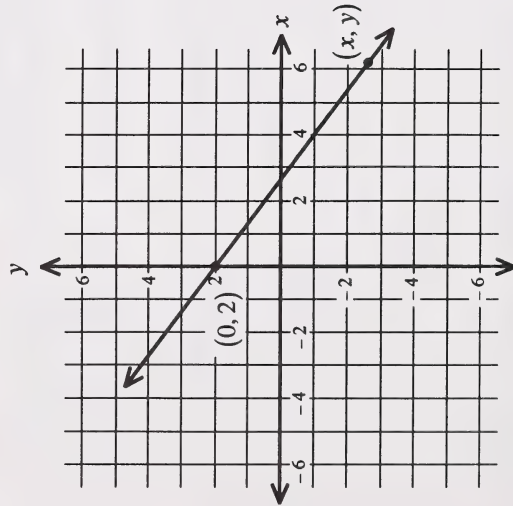
Example 4

Determine the equation of a line where the slope is $-\frac{3}{4}$ and the y-intercept is 2.

Solution:

Graph the line and let $P(x, y)$ be any point on the line.

The y-intercept of 2 means that the point $(0, 2)$ is on the line.



Substitute $m = -\frac{3}{4}$ and $(x_1, y_1) = (0, 2)$ in the point-slope formula.

$$y - 2 = -\frac{3}{4}(x - 0)$$

$$y - 2 = -\frac{3}{4}x$$

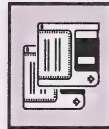
$$y - 2 + \frac{3}{4}x = -\frac{3}{4}x + \frac{3}{4}x$$

$$y - 2 + \frac{3}{4}x = 0 \quad (\text{Multiply by 4.})$$

$$4y - 8 + 3x = 0$$

$$3x + 4y - 8 = 0$$

The equation of the line is $3x + 4y - 8 = 0$.



You may use the computer program titled *Algebra Drill and Practice*,¹ Part 3, Section 3 for additional practice on finding the equation of a line, given the slope and a point on the line. Do as many questions as you feel are necessary.

¹ Algebra Drill and Practice is a title of *Conduit*.

Try the following exercises. Graph paper is provided in **Appendix B**.

Complete questions 1, 4, 5, and 6. You may do the others for more practice.

1. Determine the equation of a line when the coordinates of a point on the line and the slope are given.

a. $(-3, 2)$, $m = \frac{1}{2}$

b. $(5, -1)$, $m = 0$

c. $(7, 3)$, $m = -3$

d. $(\frac{1}{2}, -2)$, $m = -\frac{2}{3}$

2. Graph and find the equation of a line which has a slope of $\frac{3}{4}$ and a y -intercept of -2 .

3. Which of the following lines pass through the point $(2, 3)$?

a. $y = -x + 5$

b. $y = 2x - 3$

c. $4x - y = 5$

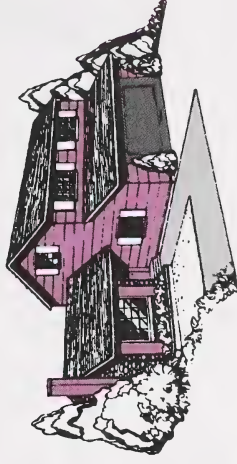
4. The x -intercept of a line is -3 and its slope is $\frac{3}{2}$.

- a. What are the coordinates of a point on the line?
- b. Determine the equation of the line.
- c. Graph the line.

5. Find the equation of a line where the slope is $-\frac{4}{3}$ and

- a. the y -intercept is 4
- b. the x -intercept is -2.5
- c. it passes through $(-4, -3)$

6. Explain why the steepness of the roof is often considered when constructing houses.



For solutions to **Activity 2**, turn to **Appendix A**, **Topic 6**.

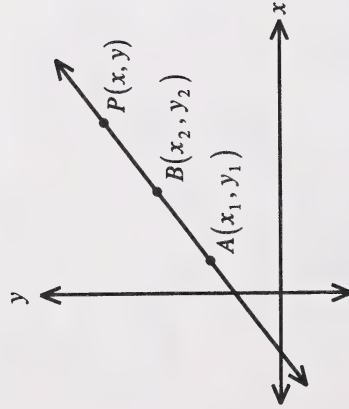
Activity 3



Determine the equation of a line passing through two given points.

A fundamental property of a straight line is the constant slope property. It states that the slope of any two line segments on the same line are equal.

From elementary geometry it is known that only one line can pass through two given points. It follows that the coordinates of the two points should determine the equation of the line passing through them.



Let the two given points be represented by $A(x_1, y_1)$ and $B(x_2, y_2)$. Let $P(x, y)$ be any other point on the line.

Substitute the coordinates $A(x_1, y_1)$ and $B(x_2, y_2)$ into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now use the point-slope formula which is $y - y_1 = m(x - x_1)$ to find the equation.

Choose $A(x_1, y_1)$ or $B(x_2, y_2)$ as the given point, use $P(x, y)$ as any point, and substitute for m , the slope you calculated.

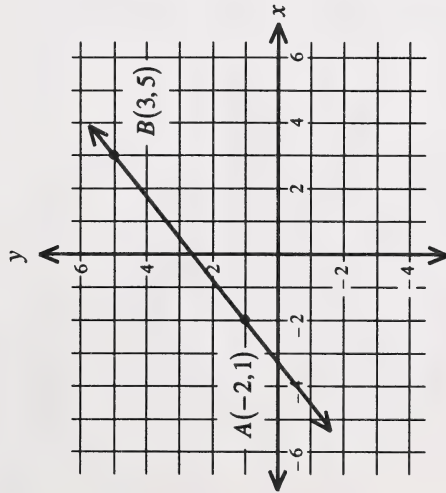
In the following examples, you will first find the slope and then use the point-slope formula to find the equation of the line.

Example 5

Find the equation of a straight line passing through the points $A(-2, 1)$ and $B(3, 5)$.

Solution:

Graph the line.



Use the slope formula to find the slope of the line segment joining the two points.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 1}{3 - (-2)} \\ &= \frac{4}{5} \end{aligned}$$

Now find the equation of the line by using the point-slope equation. Substitute one of the points into the point-slope equation, and find the equation for this line in standard form.

$$(x_1, y_1) = (-2, 1)$$

$$y - 1 = \frac{4}{5}[x - (-2)]$$

$$5(y - 1) = 4(x + 2)$$

$$5y - 5 = 4x + 8$$

$$4x - 5y + 13 = 0$$

or

$$(x_1, y_1) = (3, 5)$$

$$y - 5 = \frac{4}{5}(x - 3)$$

$$5(y - 5) = 4x - 12$$

$$5y - 25 = 4x - 12$$

$$4x - 5y + 13 = 0$$

The equation of the line is $4x - 5y + 13 = 0$.

Remember: To put an equation of a line in standard form means the x -term is first, followed by the y -term, then the constant. Making the x -term positive is the more acceptable form.

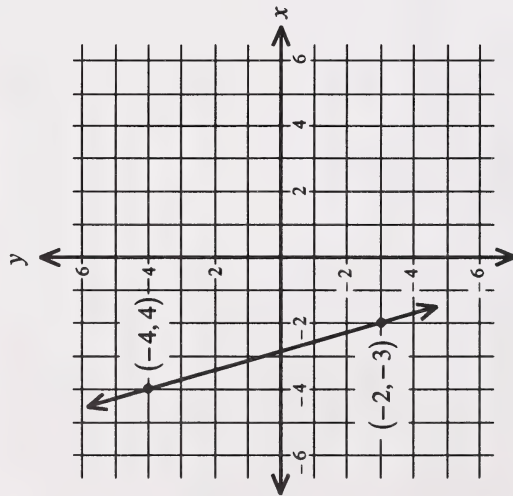


Example 6

Find the equation of the line that passes through $(-2, -3)$ and $(-4, 4)$.

Solution:

Graph the line.



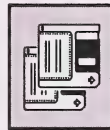
$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{4 - (-3)}{-4 - (-2)} \\
 &= \frac{4 + 3}{-4 + 2} \\
 &= \frac{-7}{2}
 \end{aligned}$$

$$[(x_1, y_1) = (-2, -3), (x_2, y_2) = (-4, 4)]$$

Now use the point-slope formula.

$$\begin{aligned}
 y - y_1 &= m(x - x_1) \\
 y - (-3) &= \frac{-7}{2}[x - (-2)] \\
 y + 3 &= \frac{-7}{2}(x + 2) \\
 2(y + 3) &= -7(x + 2) \\
 2y + 6 &= -7x - 14 \\
 7x + 2y + 20 &= 0
 \end{aligned}$$

Once you have the confidence in using the formula, you may not have to graph the line. Graphing the line helps to confirm whether the slope is positive or negative.



Use the computer program entitled *Algebra Drill and Practice*¹, Part 3, Section 4 for additional practice on finding the equation of a line given two points on the line.

¹ Algebra Drill and Practice is a title of *Conduit*.

Now try the following exercises.

Do questions 4 to 7. Do questions 1 to 3 if you need more practice.

1. Find the equation of a line passing through the following points.

- a. $(-1, 3)$ and $(2, -5)$
- b. $(-4, -4)$ and $(-8, 0)$
- c. $\left(\frac{1}{2}, 2\right)$ and $(-1, 3)$
- d. $(4, 2)$ and $(8, -1)$

2. Find the equation of the line that passes through $(3, -9)$ and cuts the y -axis at -4 .

3. The points $(3, -2)$ and $(-4, -10)$ are on a line. Determine the equation of this line.

4. Use the point-slope formula to find the equation of a line that passes through the following points.

- a. $(-6, -12)$ and $(-7, -3)$
- b. $(5, 3)$ and $(1, -4)$

5. Find the equation of a line passing through the following ordered pairs.

- a. $(4, -2)$ and $(-1, 6)$
- b. $(-5, 2)$ and $(-3, -3)$
- c. $(-3, 9)$ and $(2, -8)$
- 6. A line passes through $(-3, -\frac{1}{2})$ and has an x -intercept of -4 .

a. Find the coordinates of the point on the x -axis.

b. Graph the line.

c. Give the slope of the line.

d. Determine the equation of the line.

7. The points $Q(2, -5)$ and $R(-4, 1)$ lie on a line. If $P(x, y)$ is any point on this line, explain why $\frac{y+5}{x-2} = -1$ and $\frac{y-1}{x+4} = -1$ are true.



For solutions to **Activity 3**, turn to **Appendix A**,
Topic 6.

Activity 4



Determine the equations of horizontal and vertical lines.

Generally, when you are asked to find the equation of a line, you refer to the **constant slope property** or the **point-slope form**.

$$y - y_1 = m(x - x_1)$$

If you determine from the information given that the line will be either horizontal or vertical, then often the point-slope form will not be useful.

You may recall from the topic on graphing horizontal and vertical lines that their equations have special forms.

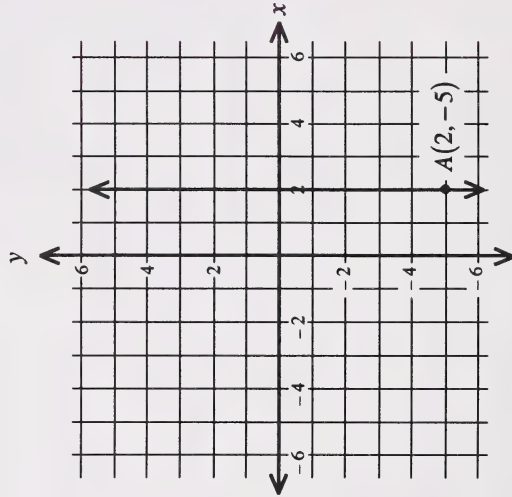
Look at some examples.

Example 7

Determine the equation of a vertical line passing through the point $A(2, -5)$.

Solution:

A vertical line means that the form of the equation will be $x = k$. Now you need to find a value for k . If the line passes through $A(2, -5)$, then x must be 2 at that point. since a vertical line must have all x -values the same, then the equation must be $x = 2$, or in general form $x - 2 = 0$.



The constant slope property states that the slope of any two line segments on the same line are equal.

In a vertical line, the y -value is any value you may wish to choose even though it is not mentioned. Some ordered pairs would be $(2, 0)$, $(2, 3)$, $(2, -1)$, $(2, 6)$, $(2, -2)$, $(2, 4)$, and $(2, -3)$.

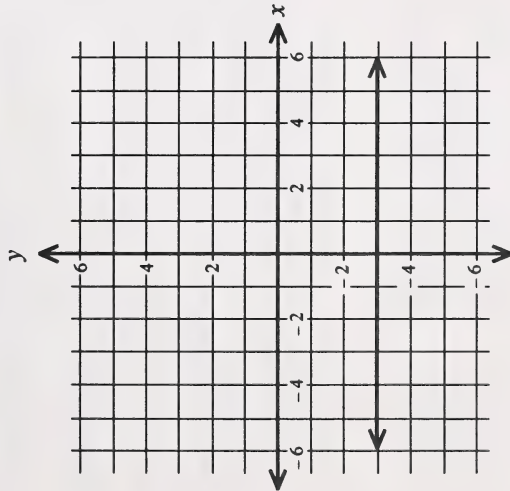
Example 8

Determine the equation of a line parallel to the x -axis and three units below it.

Solution:

Lines parallel to the x -axis are horizontal; therefore, the equation will be of the form $y = k$. Three units below implies that $k = -3$.

Therefore, the equation will be $y = -3$ or $y + 3 = 0$.



Do the following exercises.

Answer questions 2b, 3, and 4. Do the others for more practice.

- Write the equation of a line that is parallel to the y -axis.
- Determine the equation of a vertical line if it passes through the following points.
 - $(-7, -3)$
 - $(-5, 2)$
 - $(12, 5)$
- Determine the equation of a horizontal line which goes through the following points.
 - $(15, -10)$
 - $(-8, -2)$
 - $(-7, 4)$
- Explain why the slope of a vertical line is undefined.
- Why is the slope of a horizontal line zero?



For solutions to **Activity 4**, turn to **Appendix A, Topic 6**.

In a horizontal line the x -value is any value that you may wish to choose even though it is not mentioned. Some ordered pairs would be $(0, -3)$, $(4, -3)$, $(-4, -3)$, $(2, -3)$, $(5, -3)$, and $(-1, -3)$.

Activity 5



Determine the equation of a line given one point and the equation of a line parallel or perpendicular to it.

In some situations, the information required may not be obvious. You may have to obtain the information you need from what is given. As you continue, you will see how needed facts are derived from various given equations.

Study the following examples.

Example 9

A line parallel to the line $x - 2y + 4 = 0$ passes through the point $(3, -2)$. Determine the equation of this line.

Solution:

Write $x - 2y + 4 = 0$ in slope y-intercept form. The slope of this line is helpful in finding the equation of the new line.

$$\begin{array}{ll}
 x - 2y + 4 = 0 & \\
 x - 2y + 4 - x = 0 - x & \text{(Subtract } x \text{ from both sides.)} \\
 -2y + 4 = -x & \\
 -2y + 4 - 4 = -x - 4 & \text{(Subtract 4 from both sides.)} \\
 -2y = -x - 4 & \\
 \frac{-2y}{-2} = \frac{-x}{-2} - \frac{4}{-2} & \text{(Divide each term by } -2.) \\
 y = \frac{1}{2}x + 2 &
 \end{array}$$

The line $\frac{1}{2}x + 2$ has a slope of $\frac{1}{2}$. Since the required line is parallel to this line, it must also have a slope of $\frac{1}{2}$.

Now use the point-slope form to find the equation of the new line.

$$\begin{array}{l}
 y - y_1 = m(x - x_1) \\
 y - (-2) = \frac{1}{2}(x - 3) \\
 y + 2 = \frac{1}{2}(x - 3) \\
 2(y + 2) = x - 3 \\
 2y + 4 = x - 3 \\
 x - 2y - 7 = 0
 \end{array}$$

$m = \frac{1}{2}$ and $(x_1, y_1) = (3, -2)$
 The equation of the line is $x - 2y - 7 = 0$.

Parallel lines have equal slopes.
 $m_1 = m_2$

Example 10

Determine the equation of a line that passes through $(5, -1)$ and is parallel to the line joining $A(-2, -3)$ and $B(-4, 3)$.

Solution:

Graph the line AB and the new line. Find the slope of line AB . You need this slope to find the equation of the new line.

$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - (-3)}{-4 - (-2)} \\ &= \frac{3+3}{-4+2} \\ &= \frac{6}{-2} \\ &= -3 \end{aligned}$$

Since the required line is parallel to this line, it must also have a slope of -3 .

You now have a point and the slope for this line. Use the point-slope form to find its equation.

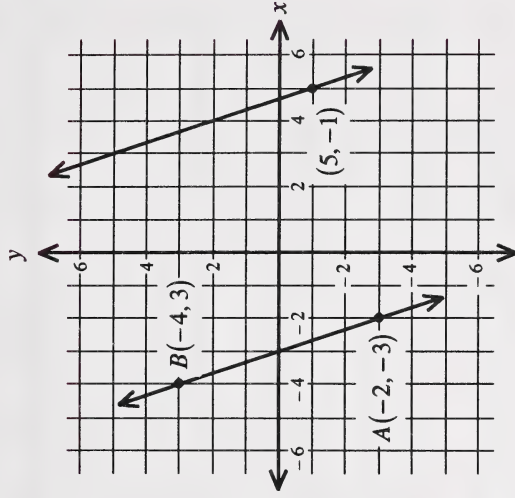
$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -3(x - 5)$$

$$y + 1 = -3x + 15$$

$$3x + y - 14 = 0$$

The equation of the line is $3x + y - 14 = 0$.



In **Topic 3** of this unit you found that if two lines are perpendicular, the slope of either line is the negative reciprocal of the slope of the other line. That is $m_1 = \frac{-1}{m_2}$ or $m_2 = \frac{-1}{m_1}$.

$$m_2 = \frac{-1}{m_1} \text{ or } m_1 = \frac{-1}{m_2}$$

Also, if two lines are perpendicular, the product of their slopes is -1 or $m_1 \times m_2 = -1$.

Go to the next example to see how you can obtain the equation of a line that is perpendicular to a given line and passes through a given point.

Example 11

Determine the equation of a line that is perpendicular to

$$3x - 4y + 7 = 0 \text{ and passes through } (-1, 4).$$

Solution:

Write the equation in slope y -intercept form. You need this slope to find the equation of the new line.

$$3x - 4y + 7 = 0 \text{ becomes } y = \frac{3}{4}x + \frac{7}{4}, \text{ where } m_1 = \frac{3}{4}.$$

Since the required line is perpendicular to the given line, its slope is $-\frac{4}{3}$ (the negative reciprocal of $\frac{3}{4}$).

The point-slope form of the equation of the line is as follows:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{4}{3}[x - (-1)]$$

$$3(y - 4) = -4(x + 1)$$

$$3y - 12 = -4x - 4$$

$$4x + 3y - 8 = 0$$

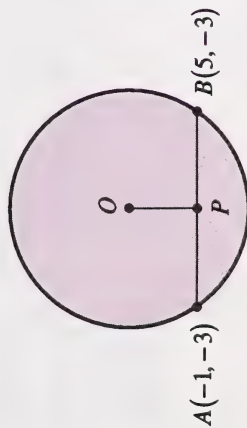
The required equation is $4x + 3y - 8 = 0$.

Remember that whenever you are asked to find the equation of a line, use the point-slope form unless the line is horizontal or vertical. You require the slope and a point in order to use this method. In cases where these are not obvious, use the skills you have acquired in this unit to find the necessary information.

Do the even-numbered questions. You may also do the odd-numbered questions if you need more practice.

1. Determine the equation of a line parallel to $2x - 2y + 3 = 0$ which passes through $(-1, 3)$.
2. Determine the equation of a line passing through $(4, 2)$ which is parallel to the line joining $A(-2, 1)$ and $B(-3, -3)$.
3. A line is parallel to the line $x = -2$ and passes through the point $(-5, 2)$. Write its equation.

4. Find the equation of a line that is perpendicular to $4y = x - 8$ and passes through $(2, -4)$.
5. Find the equation of a line which is perpendicular to $3x - 2y = 6$ and passes through $(-2, 0)$.
6. Find the equation of a line which is perpendicular to $x = 5$ and passes through $(2, -3)$.
7. A line with a y-intercept of 2 is perpendicular to the line joining $A(-2, -1)$ and $B(4, 3)$. Find the equation of the line perpendicular to $\overline{AB}$.
8. The following circle has centre O . The line joining $A(-1, -3)$ and $B(5, -3)$ is a chord. $\overline{OP}$ is the perpendicular bisector of $\overline{AB}$. Explain how you will find the equation of $\overline{OP}$.



For solutions to Activity 5, turn to **Appendix A, Topic 6**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

Consider these points.

- The equation of a line in standard form is $Ax + By + C = 0$, where A , B , and C are constants.
- The point-slope form is $y - y_1 = m(x - x_1)$. This form is used to find the equation of a line when the slope and a point on the line are given.
- The slope of parallel lines are equal.

$$m_1 = m_2$$

- If two lines are perpendicular, the slope of one line is the negative reciprocal of the slope of the other.

$$m_1 = \frac{-1}{m_2}$$

Now do the even- or odd-numbered questions.

1. Find A , B , and C in each of the following.

a. $-3x + 5y = 6$

b. $2x - 5y - 2 = 0$

c. $x + 8y = 1$

2. Write each equation in standard form.

a. $-2x + y = 5$

b. $x - 7y = -10$

3. What are the slopes of $y = -x + 8$ and $y = x - 2$? Are these lines perpendicular or parallel?

4. State whether the following pairs of lines are perpendicular, parallel, or neither.

a. $y = 2x + 5$ and $y = 2x - 7$

b. $y = x + 1$ and $y = -3x + 1$

c. $y = 4x - 2$ and $y = -\frac{1}{4}x - 8$

5. The slope of a line is $-\frac{4}{5}$. Find the equation of the line if it passes through $(-2, 4)$.

6. $A(3, -3)$ and $B(-2, -5)$ are two points on a line. Find the equation of this line.



For solutions to Extra Help, turn to **Appendix A, Topic 6**.



Extensions



Look at an equation that has fractional coefficients.

Example 12

Express the equation $\frac{1}{2}y = -\frac{2}{3}x + 2$ in

$Ax + By + C = 0$ form. Identify the values of A , B , and C .

Solution:

$$\frac{1}{2}y = -\frac{2}{3}x + 2$$

$$\frac{1}{2}y + \frac{2}{3}x = -\frac{2}{3}x + 2 + \frac{2}{3}x \quad \left(\text{Add } \frac{2}{3}x \text{ to both sides.}\right)$$

$$\frac{1}{2}y + \frac{2}{3}x = 2$$

$$\frac{1}{2}y + \frac{2}{3}x - 2 = 2 - 2$$

$$\frac{1}{2}y + \frac{2}{3}x - 2 = 0 \quad \left(\text{Subtract 2 from both sides.}\right)$$

$$\frac{2}{3}x + \frac{1}{2}y - 2 = 0$$

$$A = \frac{2}{3}, B = \frac{1}{2}, \text{ and } C = -2$$

You can change this fractional equation to an integral equation if A , B , and C must be integers.

Multiply each term of $\frac{2}{3}x + \frac{1}{2}y - 2 = 0$ by the L.C.D. which is 6.

$$6\left(\frac{2}{3}x + \frac{1}{2}y - 2\right) = 0$$
$$4x + 3y - 12 = 0$$

In this case $A = 4$, $B = 3$, and $C = -12$.

Now try the following exercises.

Do any two of the following.

Write each equation with integral coefficients; then, state the values of A , B , and C .

1. $y = -\frac{4}{5}x + \frac{1}{2}$

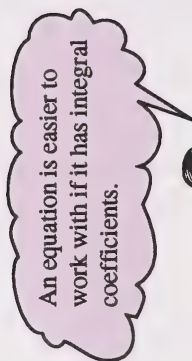
2. $-\frac{1}{4}y + \frac{1}{2}x = 3$

3. $-\frac{2}{3}x = \frac{5}{6}y - \frac{1}{3}$



For solutions to Extensions, turn to **Appendix A**,
Topic 6.

L.C.D. means **lowest common denominator**.



An equation is easier to work with if it has integral coefficients.

Unit Summary



What You Have Learned

In this unit you learned to do the following:

- Sketch linear equations in a coordinate plane using the following:
 - a table of values
 - x - and y -intercepts
 - slope and y -intercept
 - Develop and use the distance formula.
 - Derive and use the formula to find the midpoint of a line segment.
 - Develop and use the slope formula.
 - Solve problems dealing with parallel and perpendicular lines.
- Determine if points are collinear.
 - Rewrite equations in slope y -intercept form.
 - Identify m as slope and b as the y -intercept.
 - Rewrite equations in standard form.
 - Determine the equations of a line given the following:
 - slope and a point on the line
 - two points on the line
 - horizontal or vertical lines
 - one point and the equation of a perpendicular or a parallel line

You are now ready to complete the **Unit Assignment**.

Appendices



Appendix A Solutions

Review

Topic 1 Distance Between Two Points

Topic 2 Finding the Midpoint

Topic 3 Slope of a Line

Topic 4 Graphing Linear Equations

Topic 5 Slope and y -intercept

Topic 6 Equations with Unique Conditions



Appendix B Graph Paper

Grids



Appendix A Solutions



Review

1. $c^2 = a^2 + b^2$

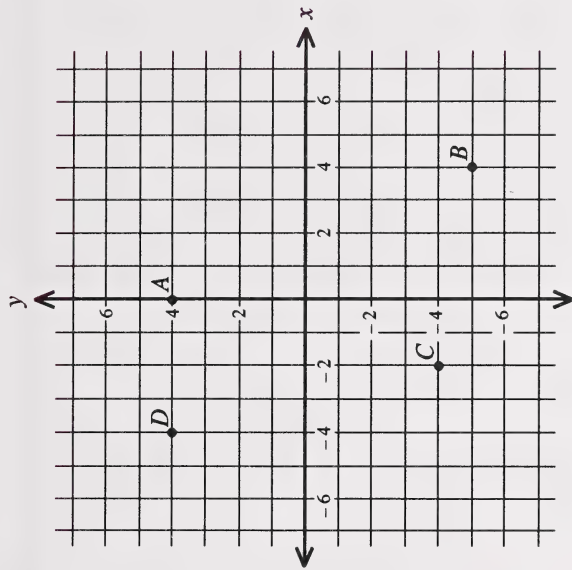
$$c^2 = 5^2 + 12^2$$

$$c^2 = 25 + 144$$

$$c^2 = 169$$

$$c = \sqrt{169}$$
$$= 13 \text{ cm}$$

2.



3. $y = 3(-3) + 4$
 $= -9 + 4$
 $= -5$



Exploring Topic 1

Activity 1

Find the distance between two points.

$$\begin{aligned} 1. \quad a. \quad AB &= |9 - 4| \text{ or } AB = |4 - 9| \\ &= |5| \\ &= 5 \end{aligned}$$

$$\begin{aligned} b. \quad CD &= |-8 - (-3)| \text{ or } CD = |-3 - (-8)| \\ &= |-5| \\ &= 5 \end{aligned}$$

$$\begin{aligned} c. \quad EF &= |-9 - 3| \text{ or } EF = |-3 - (-9)| \\ &= |-12| \\ &= 12 \end{aligned}$$

$$\begin{aligned} d. \quad GH &= |8 - (-4)| \text{ or } GH = |-4 - 8| \\ &= |12| \\ &= 12 \end{aligned}$$

$$\begin{aligned} e. \quad IJ &= |8 - 3| \text{ or } IJ = |3 - 8| \\ &= |5| \\ &= 5 \end{aligned}$$

$$\begin{aligned} f. \quad KL &= |9 - 3| \text{ or } KL = |3 - 9| \\ &= |6| \\ &= 6 \end{aligned}$$

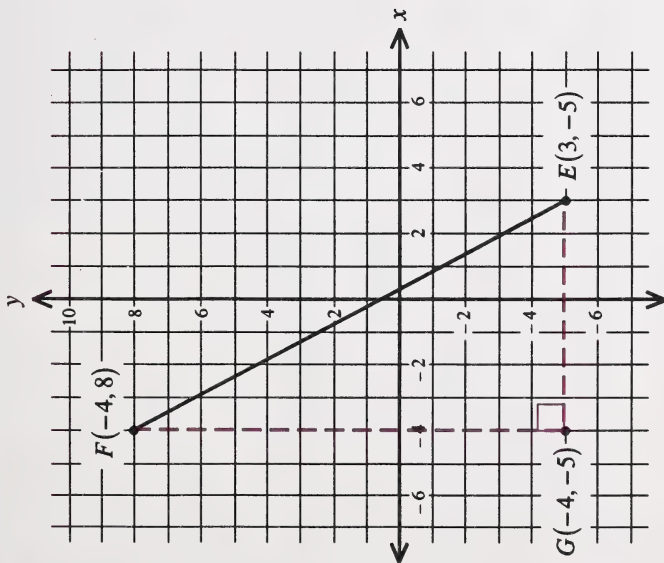
$$\begin{aligned} g. \quad MN &= |3 - (-8)| \text{ or } MN = |-8 - 3| \\ &= |11| \\ &= 11 \end{aligned}$$

$$\begin{aligned} h. \quad OP &= |7 - (-3)| \text{ or } OP = |-3 - 7| \\ &= |10| \\ &= 10 \end{aligned}$$

$$\begin{aligned} 2. \quad a. \quad AB &= |8 - (-2)| \text{ or } AB = |-2 - 8| \\ &= |10| \\ &= 10 \end{aligned}$$

$$\begin{aligned} b. \quad PQ &= |-8 - 16| \text{ or } PQ = |16 - (-8)| \\ &= |-24| \\ &= 24 \end{aligned}$$

c.



$$(EF)^2 = (EG)^2 + (FG)^2$$

$$(EF)^2 = |-4 - 3|^2 + |8 - (-5)|^2$$

$$(EF)^2 = |-7|^2 + |13|^2$$

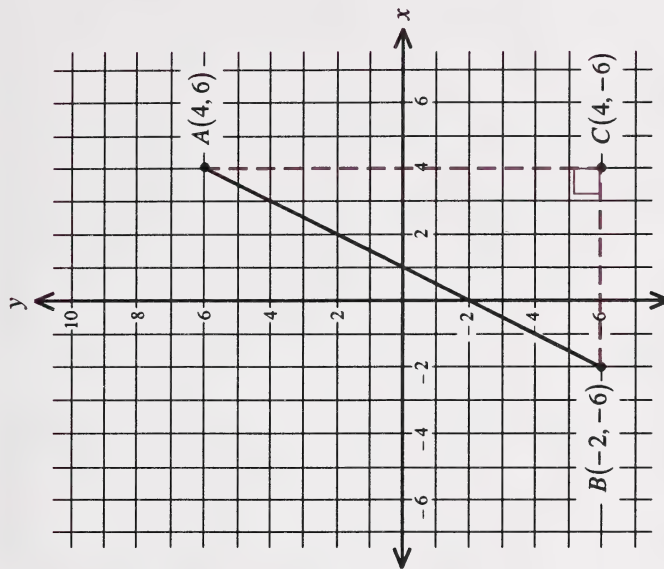
$$(EF)^2 = 49 + 169$$

$$(EF)^2 = 218$$

$$EF \doteq 14.8$$

The distance between E and F is approximately 14.8 units.

d.



$$(AB)^2 = (BC)^2 + (AC)^2$$

$$(AB)^2 = |4 - (-2)|^2 + |6 - (-6)|^2$$

$$(AB)^2 = |4 + 2|^2 + |6 + 6|^2$$

$$(AB)^2 = 36 + 144$$

$$(AB)^2 = 180$$

$$AB \doteq 13.4$$

The distance between A and B is approximately 13.4 units.

Activity 2

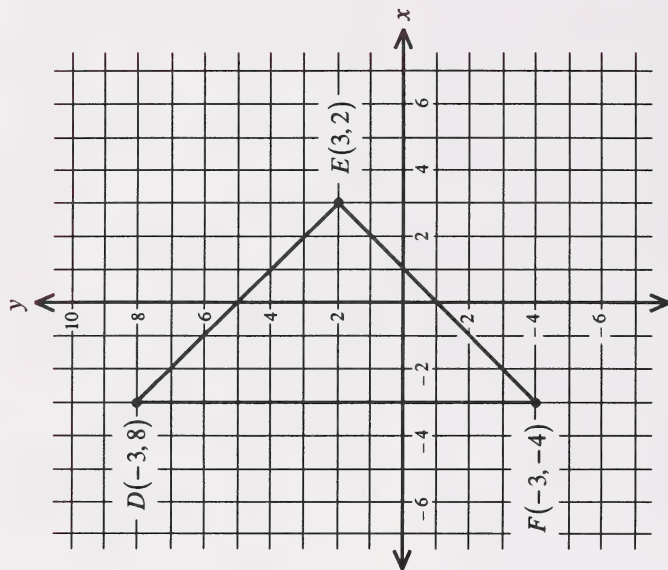
Develop and use the distance formula to solve problems.

$$\begin{aligned}
 1. \quad a. \quad & \sqrt{(5-0)^2 + (-2-6)^2} = \sqrt{(5)^2 + (-8)^2} \\
 & = \sqrt{25 + 64} \\
 & = \sqrt{89} \\
 & \doteq 9.4
 \end{aligned}$$

$$\begin{aligned}
 b. \quad & \sqrt{[-4-(-3)]^2 + [7-(-8)]^2} = \sqrt{(-4+3)^2 + (7+8)^2} \\
 & = \sqrt{(-1)^2 + (15)^2} \\
 & = \sqrt{1 + 225} \\
 & = \sqrt{226} \\
 & \doteq 15.0
 \end{aligned}$$

$$\begin{aligned}
 c. \quad & \sqrt{(3-1)^2 + [0-(-2.5)]^2} = \sqrt{(2)^2 + (2.5)^2} \\
 & = \sqrt{4 + 6.25} \\
 & = \sqrt{10.25} \\
 & \doteq 3.2
 \end{aligned}$$

2.



$$\begin{aligned}
 DE &= \sqrt{[3 - (-3)]^2 + (2 - 8)^2} \\
 &= \sqrt{(3+3)^2 + (-6)^2} \\
 &= \sqrt{(6)^2 + (-6)^2} \\
 &= \sqrt{36 + 36} \\
 &= \sqrt{72} \\
 &\doteq 8.5
 \end{aligned}$$

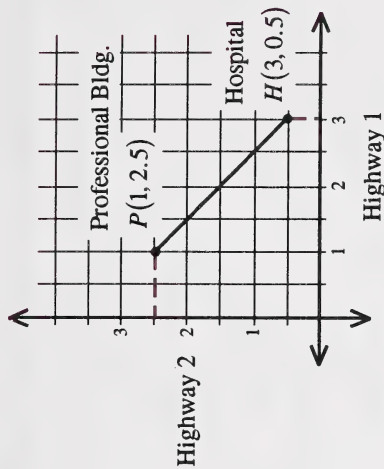
$$\begin{aligned}
 FE &= \sqrt{[3 - (-3)]^2 + [2 - (-4)]^2} \\
 &= \sqrt{(3+3)^2 + (2+4)^2} \\
 &= \sqrt{(6)^2 + (6)^2} \\
 &= \sqrt{36 + 36} \\
 &= \sqrt{72} \\
 &\doteq 8.5
 \end{aligned}$$

$$\begin{aligned}
 DF &= \sqrt{[-3 - (-3)]^2 + (-4 - 8)^2} \quad \text{or} \quad DF = |8 - (-4)| \\
 &= 12
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{(-3+3)^2 + (-12)^2} \\
 &= \sqrt{0 + 144} \\
 &= 12
 \end{aligned}$$

Since $DE = FE$, $\triangle DEF$ is an isosceles triangle.

3.



$$\begin{aligned}
 PH &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(3-1)^2 + (0.5-2.5)^2} \\
 &= \sqrt{(2)^2 + (-2)^2} \\
 &= \sqrt{4+4} \\
 &= \sqrt{8} \\
 &\doteq 3 \text{ km}
 \end{aligned}$$

The two buildings are about 3 km apart.

The total distance along the highways is as follows:

$$\begin{aligned}
 d &= 1 \text{ km} + 2.5 \text{ km} + 3 \text{ km} + 0.5 \text{ km} \\
 &= 7 \text{ km}
 \end{aligned}$$

Extra Help

- a. The two bars on either side of -7 in $|-7|$ means absolute value.

b. The absolute value of a positive or negative number is positive.

c. Length is always a positive number.

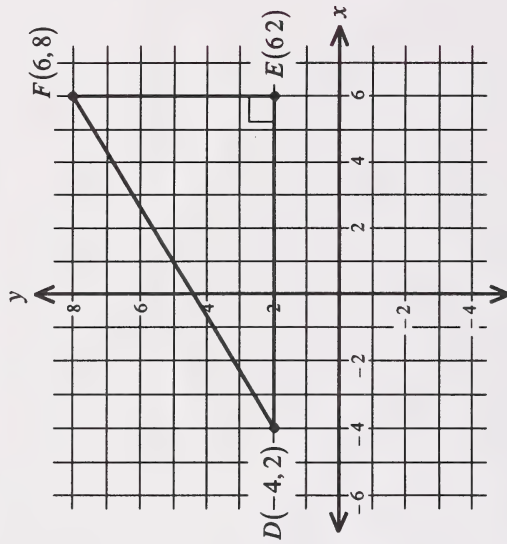
d. If $d^2 = e^2 + f^2$ represents the Pythagorean theorem, then $\angle D$ is the right angle in $\triangle DEF$.

- a. $(x_1, y_1) = (-2, 5)$
 $(x_2, y_2) = (-3, -3)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[-3 - (-2)]^2 + (-3 - 5)^2} \\
 &= \sqrt{(-1)^2 + (-8)^2} \\
 &= \sqrt{1 + 64} \\
 &= \sqrt{65} \\
 &\approx 8.1
 \end{aligned}$$

The distance between A and B is approximately 8.1 units.

- Draw $\overline{DF}$ on a coordinate plane. Complete the right triangle DEF . The coordinates of point E are $(6, 2)$.



The length of $\overline{EF}$ is $|8 - 2| = 6$.

The length of $\overline{DE}$ is $|6 - (-4)| = 10$.

$$(DF)^2 = (DE)^2 + (FE)^2$$

$$(DF)^2 = 10^2 + 6^2$$

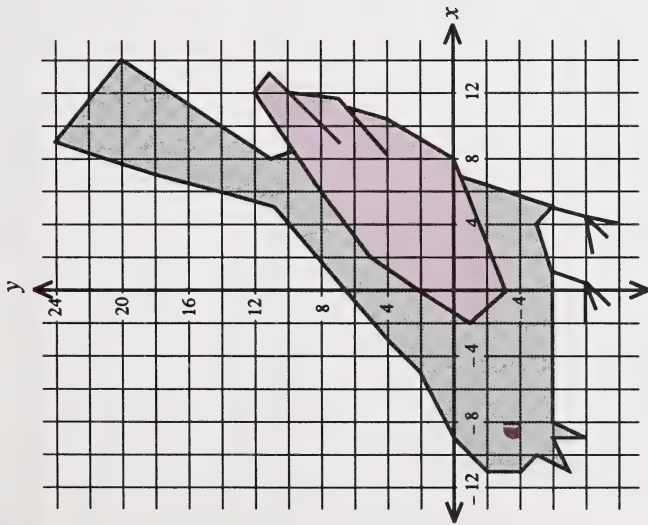
$$(DF)^2 = 100 + 36$$

$$(DF)^2 = 136$$

$$DF \approx 11.7$$

The length of $\overline{DF}$ is approximately 11.7 units.

3.



Extensions

1. a. $A\left(-4\frac{1}{3}, -6\frac{1}{2}\right)$ $B\left(5\frac{1}{2}, -1\frac{3}{4}\right)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left[5\frac{1}{2} - \left(-4\frac{1}{3}\right)\right]^2 + \left[-1\frac{3}{4} - \left(-6\frac{1}{2}\right)\right]^2} \\
 &= \sqrt{\left(5\frac{1}{2} + 4\frac{1}{3}\right)^2 + \left(-1\frac{3}{4} + 6\frac{1}{2}\right)^2} \\
 &= \sqrt{\left(5\frac{3}{6} + 4\frac{2}{6}\right)^2 + \left(-1\frac{3}{4} + 6\frac{2}{4}\right)^2} \\
 &= \sqrt{\left(9\frac{5}{6}\right)^2 + \left(4\frac{3}{4}\right)^2} \\
 &= \sqrt{\left(\frac{59}{6}\right)^2 + \left(\frac{19}{4}\right)^2} \\
 &= \sqrt{(9.8\bar{3})^2 + (4.75)^2} \\
 &= \sqrt{96.694\ 444\ 44 + 22.5625} \\
 &= \sqrt{119.257\ 444\ 44} \\
 &= 10.920\ 505\ 69 \\
 &\approx 10.92
 \end{aligned}$$

The distance between points A and B is approximately 10.92 units.

b. $A = \left(6\frac{1}{5}, 3\frac{1}{4}\right)$ $B = \left(-1\frac{1}{8}, -6\frac{5}{9}\right)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(-1\frac{1}{8} - 6\frac{1}{5}\right)^2 + \left(-6\frac{5}{9} - 3\frac{1}{4}\right)^2} \\
 &= \sqrt{\left(-1\frac{5}{40} - 6\frac{8}{40}\right)^2 + \left(-6\frac{20}{36} - 3\frac{9}{36}\right)^2} \\
 &= \sqrt{\left(-7\frac{13}{40}\right)^2 + \left(-9\frac{29}{36}\right)^2} \\
 &= \sqrt{\left(\frac{-293}{40}\right)^2 + \left(\frac{-353}{36}\right)^2} \\
 &= \sqrt{(-7.325)^2 + (-9.805)^2} \\
 &\doteq \sqrt{53.655\,625 + 96.148\,919\,75} \\
 &\doteq \sqrt{149.804\,5448} \\
 &\doteq 12.239\,466\,69 \\
 &\doteq 12.24
 \end{aligned}$$

The distance between points A and B is approximately 12.24 units.

c. $A(-4.3, -6.5)$ and $B(5.5, -1.75)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[5.5 - (-4.3)]^2 + [-1.75 - (-6.5)]^2} \\
 &= \sqrt{(9.8)^2 + (4.75)^2} \\
 &= \sqrt{96.04 + 22.5625} \\
 &= \sqrt{118.6025} \\
 &\doteq 10.890\,477\,49 \\
 &\doteq 10.89
 \end{aligned}$$

The distance between points A and B is about 10.89 units. Compare this answer with question 1.

Because -4.3 is almost $-4\frac{1}{3}$ or $-4\frac{1}{3}$, -6.5 is equal to $-6\frac{1}{2}$, 5.5 is equal to $5\frac{1}{2}$, and -1.75 is equal to $-1\frac{3}{4}$, you would expect the answers to be very close and they are. One answer is about 10.92 units and the other is about 10.89 units.

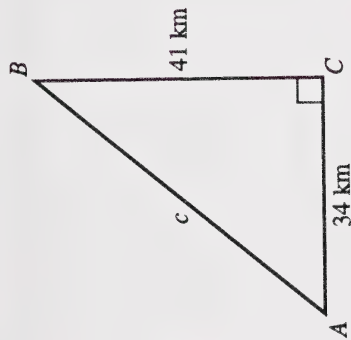
d. $A(6.2, 3.25)$ and $B(-1.125, -6.56)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1.125 - 6.2)^2 + (-6.56 - 3.25)^2} \\ &= \sqrt{(-7.325)^2 + (-9.81)^2} \\ &= \sqrt{53.655625 + 96.2361} \\ &\doteq \sqrt{149.891725} \\ &\doteq 12.24302761 \\ &\doteq 12.24 \end{aligned}$$

The distance between points A and B is about 12.24 units.
Compare this answer with question 2.

Because 6.2 is equal to $6\frac{1}{5}$, 3.25 is equal to $3\frac{1}{4}$, -1.125 is equal to $-1\frac{1}{8}$, and 6.56 is almost $6\frac{5}{9}$ or $6.\bar{5}$, you would expect the answers to be very close. In fact they are the same. Both answers are about 12.24 units.

2.



Use the Pythagorean theorem to find the distance Joe walks.

$$c^2 = a^2 + b^2$$

$$c^2 = 41^2 + 34^2$$

$$c^2 = 1681 + 1156$$

$$c^2 = 2837$$

$$c = \sqrt{2837}$$

$$\doteq 53.26349594$$

$$\doteq 53.3 \text{ km (rounded to one decimal place)}$$

Add AC and CB to find the distance that Ernie runs.

$$\begin{aligned} AC + CB &= 34 \text{ km} + 41 \text{ km} \\ &= 75 \text{ km} \end{aligned}$$

Use the formula $t = \frac{d}{r}$ to find the time it takes Joe to walk 53.3 km at 5 km/h.

$$\begin{aligned} t &= \frac{d}{r} \\ &= \frac{53.3}{5} \\ &\doteq 10.66 \end{aligned}$$

$$\doteq 10.7 \quad (\text{rounded to one decimal place})$$

It takes Joe about 10.7 h to walk from Point A to Point B.

Use the formula $t = \frac{d}{r}$ to find the time it takes Ernie to run 75 km at 7.5 km/h.

$$\begin{aligned} t &= \frac{d}{r} \\ &= \frac{75}{7.5} \\ &= 10 \text{ h} \end{aligned}$$

It takes Ernie 10 h to run from Point A to Point C and then on to Point B.

It would take Joe $10.7 - 10 \doteq 0.7$ h longer to get to Point B than it would take Ernie. Ernie would be at Point B first and would be waiting for about $0.7 \times 60 \doteq 42$ minutes for Joe to arrive.



Exploring Topic 2

Activity 1

Develop and use the midpoint formula for finding the midpoint of a line segment.

$$\begin{aligned} 1. \quad \text{midpoint of } \overline{AC} &= \left(\frac{12+0}{2}, \frac{-2+10}{2} \right) \\ &= \left(\frac{12}{2}, \frac{8}{2} \right) \\ &= (6, 4) \end{aligned}$$

Therefore, B is the midpoint of $\overline{AC}$.

$$\begin{aligned} 2. \quad \text{a.} \quad \frac{x_1 + x_2}{2} &= \frac{7+11}{2} & \frac{y_1 + y_2}{2} &= \frac{5+(-2)}{2} \\ &= \frac{18}{2} & &= \frac{3}{2} \\ &= 9 & &= 1\frac{1}{2} \end{aligned}$$

The midpoint is $(9, 1\frac{1}{2})$.

$$\begin{aligned} \text{b. } \frac{x_1 + x_2}{2} &= \frac{6 + (-10)}{2} \\ &= \frac{-4}{2} \\ &= -2 \end{aligned}$$

The midpoint is $(-2, 9)$.

$$\begin{aligned} \text{c. } \frac{x_1 + x_2}{2} &= \frac{-5 + (-8)}{2} \\ &= \frac{-13}{2} \\ &= -6\frac{1}{2} \end{aligned}$$

The midpoint is $(-6\frac{1}{2}, -7\frac{1}{2})$.

$$\begin{aligned} \text{d. } \frac{x_1 + x_2}{2} &= \frac{83 + 45}{2} \\ &= \frac{128}{2} \\ &= 64 \end{aligned}$$

The midpoint is $(64, 2)$.

$$\begin{aligned} \text{3. } \frac{x_1 + x_2}{2} &= \frac{-5 + 9}{2} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \frac{y_1 + y_2}{2} &= \frac{-9 + (-5)}{2} \\ &= \frac{-14}{2} \\ &= -7 \end{aligned}$$

The midpoint of $\overline{MN}$ is $(2, -7)$.

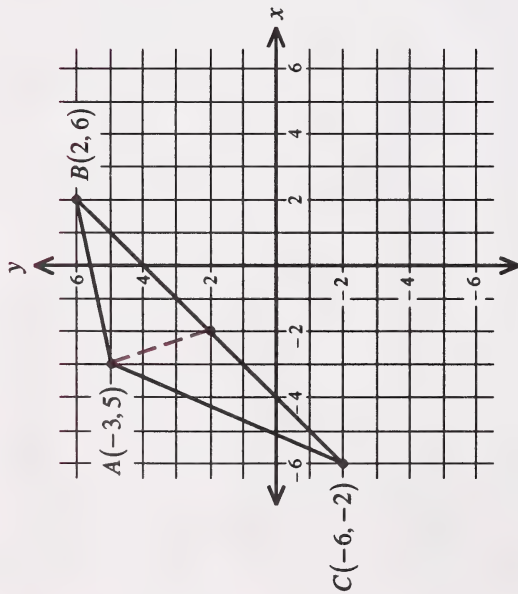
Since $(2, -7)$ is the midpoint of $\overline{MN}$, it must be equidistant from points M and N .

Activity 2

Solve problems that involve the use of the midpoint formula.

- $\overline{BC}$ is the side opposite $\angle A$. First, you must find the midpoint of $\overline{BC}$.

$$\begin{aligned} \text{midpoint of } \overline{BC} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2 + (-6)}{2}, \frac{6 + (-2)}{2} \right) \\ &= \left(\frac{-4}{2}, \frac{4}{2} \right) \\ &= (-2, 2) \end{aligned}$$



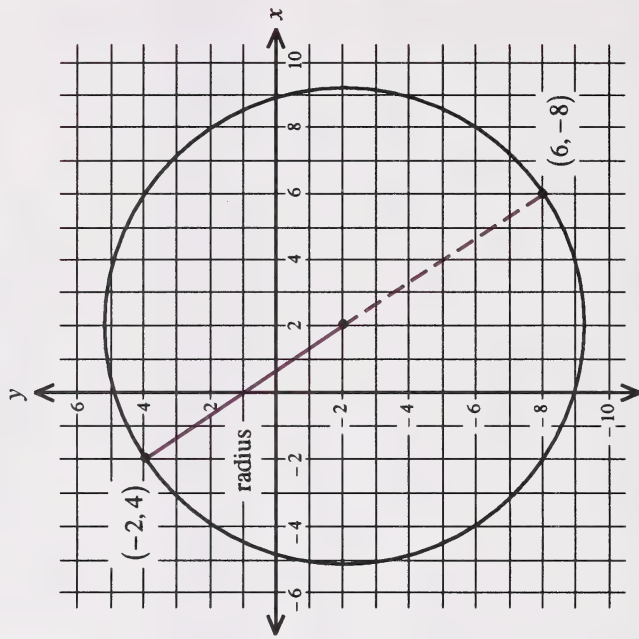
Now use the distance formula to find the length of A to point $(-2, 2)$.

$$\begin{aligned}
 \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} &= \sqrt{[-3 - (-2)]^2 + [5 - (2)]^2} \\
 &= \sqrt{(-3 + 2)^2 + (5 - 2)^2} \\
 &= \sqrt{(-1)^2 + (3)^2} \\
 &= \sqrt{1 + 9} \\
 &= \sqrt{10} \\
 &\approx 3.2
 \end{aligned}$$

Therefore, the distance from A to the midpoint of the opposite side is about 3.2 units.

2. The centre of the circle is at the midpoint of its diameter.

$$\begin{aligned}
 \text{centre of circle} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{(-2) + 6}{2}, \frac{4 + (-8)}{2} \right) \\
 &= \left(\frac{4}{2}, \frac{-4}{2} \right) \\
 &= (2, -2)
 \end{aligned}$$



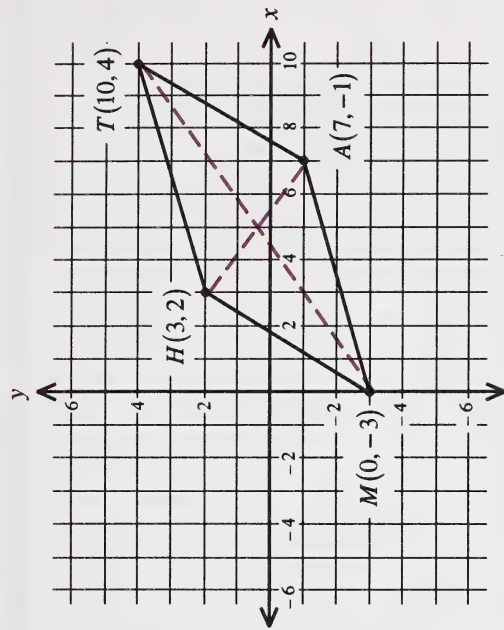
Use the distance formula to find the length of the radius.

3.

$$\begin{aligned}
 r &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\
 &= \sqrt{(-2 - 2)^2 + [4 - (-2)]^2} \\
 &= \sqrt{(-4)^2 + (6)^2} \\
 &= \sqrt{16 + 36} \\
 &= \sqrt{52} \\
 &\doteq 7.2
 \end{aligned}$$

You can use either endpoint of the diameter and the centre of the circle to find its radius.

The radius of the circle is approximately 7.2 units.



The diagonals of the parallelogram are $\overline{HA}$ and $\overline{MT}$.

$$\begin{aligned}
 \text{midpoint of } \overline{HA} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{3+7}{2}, \frac{2+(-1)}{2} \right) \\
 &= \left(\frac{10}{2}, \frac{1}{2} \right) \\
 &= \left(5, \frac{1}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{midpoint of } \overline{TM} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{10+0}{2}, \frac{4+(-3)}{2} \right) \\
 &= \left(\frac{10}{2}, \frac{1}{2} \right) \\
 &= \left(5, \frac{1}{2} \right)
 \end{aligned}$$

The midpoints of the diagonals are the same and the diagonals of the parallelogram intersect at their midpoints.

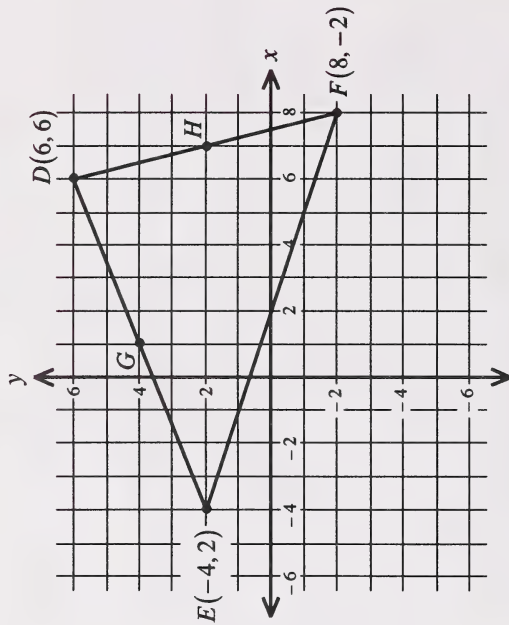
4. Find the coordinates of G and H .

G is the midpoint of $\overline{DE}$.

$$\begin{aligned}
 \text{midpoint of } \overline{DE} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{6-4}{2}, \frac{6+2}{2} \right) \\
 &= \left(\frac{2}{2}, \frac{8}{2} \right) \\
 &= (1, 4)
 \end{aligned}$$

H is the midpoint of $\overline{DF}$

$$\begin{aligned}
 \text{midpoint of } \overline{DF} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{6+8}{2}, \frac{6-2}{2} \right) \\
 &= \left(\frac{14}{2}, \frac{4}{2} \right) \\
 &= (7, 2)
 \end{aligned}$$



$$\begin{aligned}
 GH &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\
 &= \sqrt{(1 - 7)^2 + (4 - 2)^2} \\
 &= \sqrt{(-6)^2 + (2)^2} \\
 &= \sqrt{40} \\
 &= 2\sqrt{10} \\
 &\approx 6.3
 \end{aligned}$$

$$\begin{aligned}
 EF &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\
 &= \sqrt{(-4 - 8)^2 + [2 - (-2)]^2} \\
 &= \sqrt{(-12)^2 + (4)^2} \\
 &= \sqrt{160} \\
 &= 4\sqrt{10} \\
 &\approx 12.6
 \end{aligned}$$

The length of $\overline{GH}$ is half the length of $\overline{EF}$.

Extra Help

1. a. False

The midpoint is $(4, 1)$.

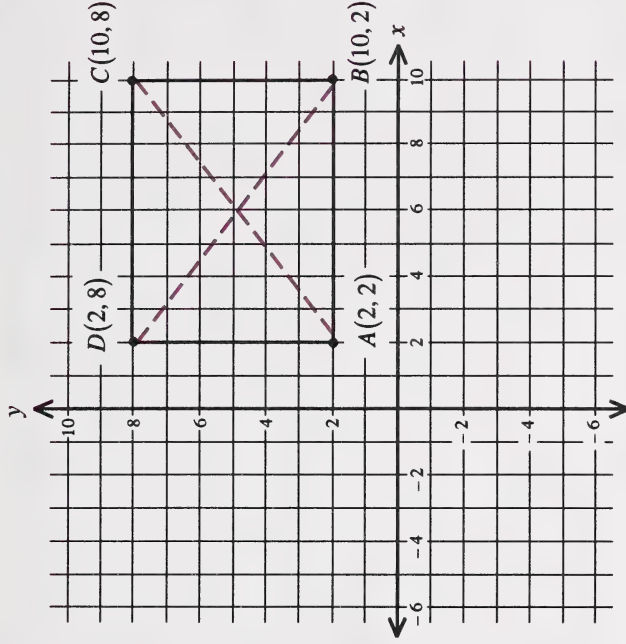
- c. True

- b. True

- d. False

The midpoint is $(6, 2)$.

2.



Midpoint of $\overline{AC}$

$$\begin{aligned}
 \frac{x_1 + x_2}{2} &= \frac{2 + 10}{2} \\
 &= \frac{12}{2} \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 \frac{y_1 + y_2}{2} &= \frac{2 + 8}{2} \\
 &= \frac{10}{2} \\
 &= 5
 \end{aligned}$$

The midpoint of $\overline{AC}$ is $(6, 5)$.

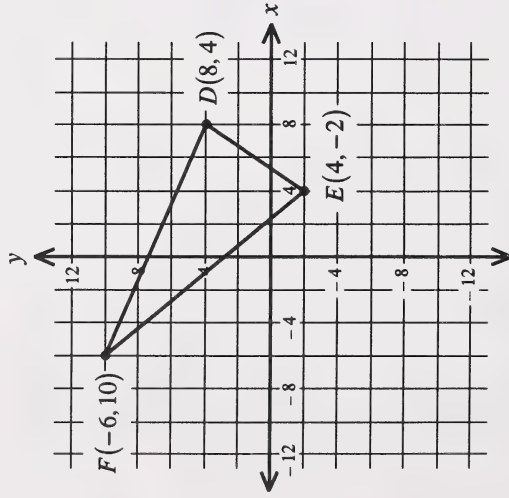
Midpoint of $\overline{DB}$

$$\begin{aligned}\frac{x_1 + x_2}{2} &= \frac{2+10}{2} \\ &= \frac{12}{2} \\ &= 6\end{aligned}$$

The midpoint of $\overline{DB}$ is $(6, 5)$.

Both diagonals have $(6, 5)$ as their midpoint. Therefore, they bisect each other.

3.



Midpoint of $\overline{DE}$

$$\begin{aligned}\frac{x_1 + x_2}{2} &= \frac{8+4}{2} \\ &= 6\end{aligned}$$

The midpoint of $\overline{DE}$ is $(6, 1)$.

Midpoint of $\overline{EF}$

$$\begin{aligned}\frac{x_1 + x_2}{2} &= \frac{4+(-6)}{2} \\ &= -1\end{aligned}$$

The midpoint of $\overline{EF}$ is $(-1, 4)$.

Midpoint of $\overline{FD}$

$$\begin{aligned}\frac{x_1 + x_2}{2} &= \frac{-6+8}{2} \\ &= 1\end{aligned}$$

The midpoint of $\overline{FD}$ is $(1, 7)$.

Extensions

$$\begin{aligned}1. \text{ midpoint of } \overline{MN} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-6+8}{2}, \frac{2+(-12)}{2} \right) \\ &= \left(\frac{2}{2}, \frac{-10}{2} \right) \\ &= (1, -5)\end{aligned}$$

Call this point Q .



Now find the midpoint of $\overline{MQ}$ and the midpoint of $\overline{QN}$.

$$\begin{aligned}\text{midpoint of } \overline{MQ} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-6+1}{2}, \frac{2-5}{2} \right) \\ &= \left(\frac{-5}{2}, \frac{-3}{2} \right) \text{ or } (-2.5, -1.5)\end{aligned}$$

$$\begin{aligned}\text{midpoint of } \overline{QN} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{1+8}{2}, \frac{-5-12}{2} \right) \\ &= \left(\frac{9}{2}, \frac{-17}{2} \right) \text{ or } (4.5, -8.5)\end{aligned}$$

The coordinates of the three points are $(1, -5)$, $(-2.5, -1.5)$, and $(4.5, -8.5)$.

$$2. \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (-3, 6)$$

Let $(x_1, y_1) = (8, 10)$.



$$\begin{aligned}\frac{8 + x_2}{2} &= -3 & \frac{10 + y_2}{2} &= 6 \\ 8 + x_2 &= -6 & 10 + y_2 &= 12 \\ x_2 &= -6 - 8 & y_2 &= 12 - 10 \\ &= -14 & &= 2\end{aligned}$$

The coordinates of B are $(-14, 2)$.

3. Find the coordinates of point B .

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (4, 2)$$

Let $(x_1, y_1) = (2, 8)$.

$$\begin{aligned}\frac{2 + x_2}{2} &= 4 & \frac{8 + y_2}{2} &= 2 \\ 2 + x_2 &= 8 & 8 + y_2 &= 4 \\ x_2 &= 8 - 2 & y_2 &= 4 - 8 \\ &= 6 & &= -4\end{aligned}$$

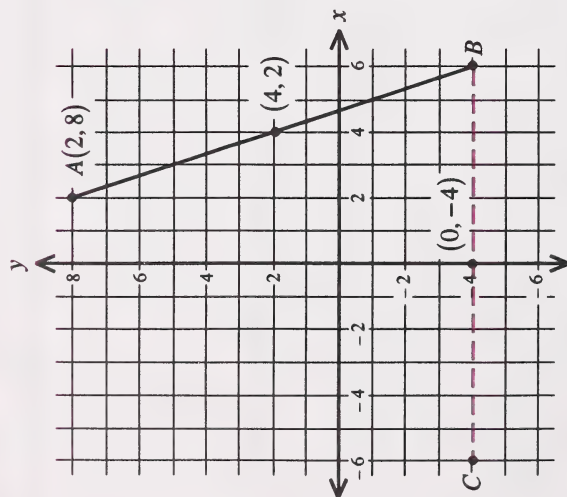
The coordinates of point B are $(6, -4)$.



Exploring Topic 3

Activity 2

Develop and apply the formula for slope.



Use point B and the midpoint $(0, -4)$ to locate point C .

Let $(x_1, y_1) = (6, 4)$.

$$\frac{x_1 + x_2}{2} = 0 \quad \text{and} \quad \frac{y_1 + y_2}{2} = -4$$

$$\frac{6 + x_2}{2} = 0 \quad \frac{-4 + y_2}{2} = -4$$

$$6 + x_2 = 0 \quad -4 + y_2 = -8$$

$$\begin{aligned} x_2 &= -6 \\ y_2 &= -8 + 4 \\ &= -4 \end{aligned}$$

The coordinates of point C are $(-6, -4)$.

$$\begin{aligned} 1. \quad a. \quad m_{\overline{IJ}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{9 - 2}{-1 - (-7)} \\ &= \frac{7}{-6} \end{aligned}$$

$$\begin{aligned} b. \quad m_{\overline{KL}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 4}{9 - 2} \\ &= \frac{2}{7} \end{aligned}$$

$$\begin{aligned} c. \quad m_{\overline{MN}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - 0}{-6 - (-3)} \\ &= \frac{-8}{-3} \\ &= \frac{8}{3} \end{aligned}$$

$$\begin{aligned} d. \quad m_{\overline{PQ}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - (-7)}{8 - 0} \\ &= \frac{3}{8} \end{aligned}$$

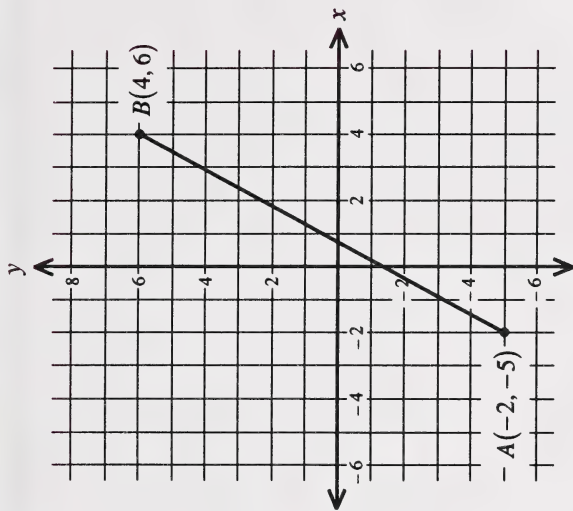
$$\begin{aligned}
 2. \quad a. \quad m_{\overline{CD}} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{2 - 6}{0 - (-8)} \\
 &= \frac{-4}{8} \\
 &= \frac{-1}{2}
 \end{aligned}$$

$$\begin{aligned}
 b. \quad m_{\overline{GH}} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-8 - (-1)}{5 - 3} \\
 &= \frac{-7}{2} \\
 &= -\frac{7}{2}
 \end{aligned}$$

$$\begin{aligned}
 c. \quad m_{\overline{AB}} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{3 - 7}{7 - 3} \\
 &= \frac{-4}{4} \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 d. \quad m_{\overline{EF}} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-7 - (-2)}{-3 - (-7)} \\
 &= \frac{-5}{4} \\
 &= -\frac{5}{4}
 \end{aligned}$$

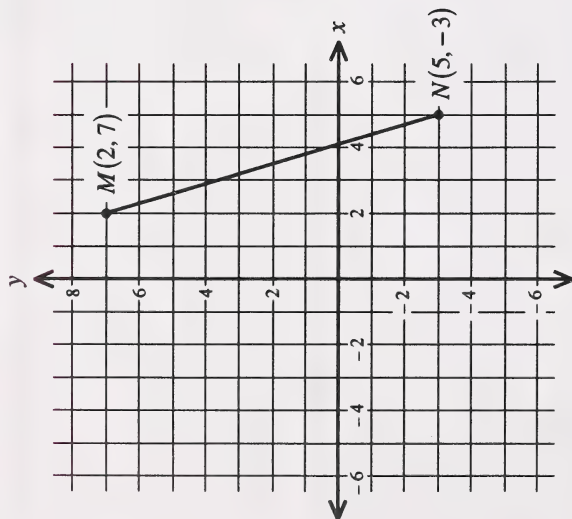
3. a. All the slopes in question 1 are positive.
- b. All the segments of question 1 rise to the right.
4. a. All the slopes in question 2 are negative.
- b. All the segments of question 2 fall toward the right (or rise toward the left).



$$\begin{aligned}
 m_{\overline{AB}} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{6 - (-5)}{4 - (-2)} \\
 &= \frac{11}{6}
 \end{aligned}$$

The slope rises to the right.

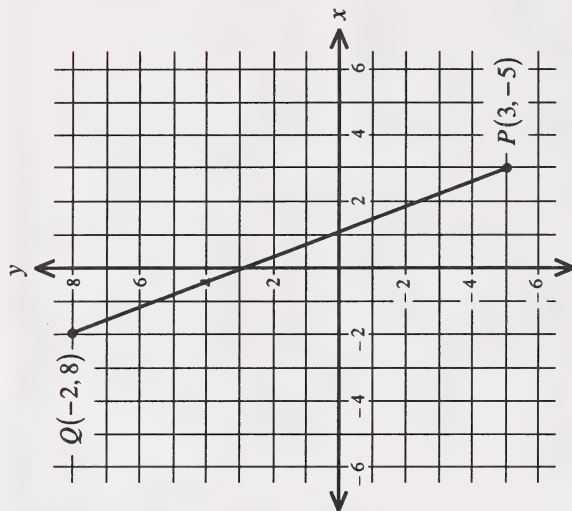
b.



$$\begin{aligned} m_{\overline{MN}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 7}{5 - 2} \\ &= \frac{-10}{3} \end{aligned}$$

The slope falls to the right.

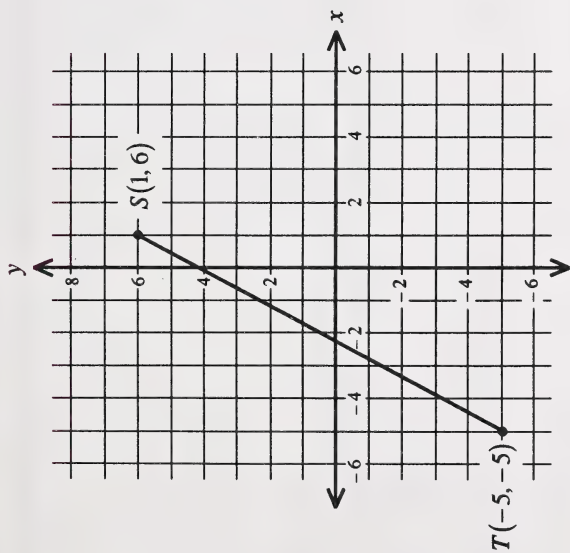
c.



$$\begin{aligned} m_{\overline{PQ}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 8}{3 - (-2)} \\ &= \frac{-13}{5} \end{aligned}$$

The slope falls to the right.

d.

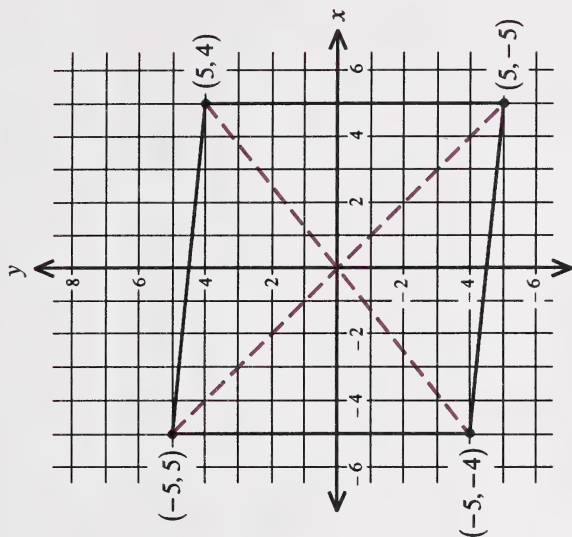


$$\begin{aligned} m_{\overline{ST}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 6}{-5 - 1} \\ &= \frac{-11}{-6} \\ &= \frac{11}{6} \end{aligned}$$

The slope rises to the right.

6. Undefined.

7.



Find the slope of the diagonal with endpoints $(5, 4)$ and $(-5, -4)$.

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{-4 - 4}{-5 - 5} \\ &= \frac{-8}{-10} \\ &= \frac{4}{5} \end{aligned}$$

Find the slope of the other diagonal with endpoints $(-5, 5)$ and $(5, -5)$.

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{-5 - 5}{5 - (-5)} \\ &= \frac{-10}{10} \\ &= -1\end{aligned}$$

$$\begin{aligned}8. \text{ slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{20}{12} \\ &= \frac{5}{3}\end{aligned}$$

The slope of the track is $\frac{5}{3}$.

$$9. \text{ slope} = \frac{\text{rise}}{\text{run}}$$

Find the ratio of the vertical distance (the height of the roof) and horizontal distance (the distance from the foot of the ladder to the building).

10. a.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ \frac{-5}{3} &= \frac{7 - (-3)}{x - 3} \\ -5(x - 3) &= 3(10) \\ -5x + 15 &= 30 \\ -5x &= 30 - 15 \\ -5x &= 15 \\ x &= \frac{15}{-5} \\ &= -3\end{aligned}$$

b.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ 2 &= \frac{-10 - x}{-4 - 4} \\ 2(-8) &= -10 - x \\ -10 - x &= 2(-8) \\ -10 - x &= -16 \\ -x &= -16 + 10 \\ -x &= -6 \\ x &= 6\end{aligned}$$

11.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ 0 &= \frac{7 - k}{7 - (-3)} \\ 7 - k &= 0 \\ k &= 7\end{aligned}$$

Activity 3

Identify and use the relationship between slopes of parallel lines.

1. a. Since $m_1 = m_2$, the lines are parallel.
- b. Since $m_1 = m_2$, the lines are parallel.

c. Since $m_1 \neq m_2$, the lines are not parallel.

d. Since $m_1 \neq m_2$, the lines are not parallel.

$$\begin{aligned} 2. \quad a. \quad m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-7}{8-9} \\ &= \frac{-6}{-1} \\ &= 6 \end{aligned}$$

Since $m_1 \neq m_2$, the lines are not parallel.

$$\begin{aligned} b. \quad m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5-(-3)}{-1-8} \\ &= \frac{8}{-9} \end{aligned}$$

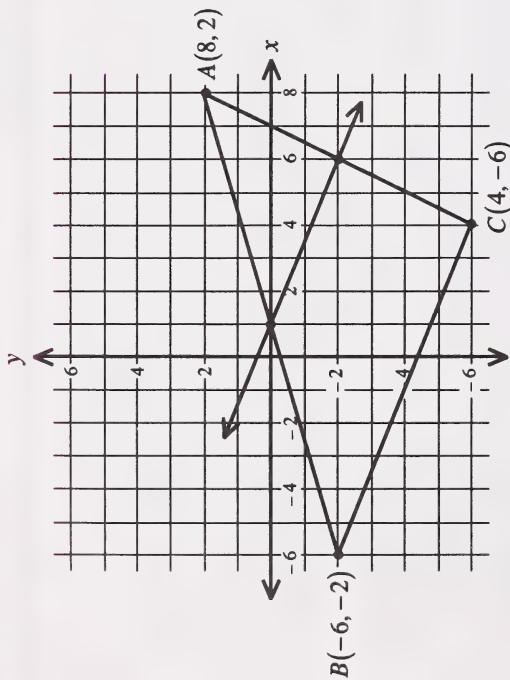
Since $m_1 = m_2$, the lines are parallel.

$$\begin{aligned} 3. \quad \text{slope of } \overline{EF} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{10-(-4)}{6-0} \\ &= \frac{14}{6} \\ &= \frac{7}{3} \end{aligned}$$

$$\begin{aligned} \text{slope of } \overline{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-p}{5-6} \\ &= \frac{3-p}{-1} \end{aligned}$$

$$\begin{aligned} \text{Since } \overline{AB} \parallel \overline{EF}, m_{\overline{AB}} &= m_{\overline{EF}} \\ \frac{3-p}{-1} &= \frac{14}{6} \\ 6(3-p) &= 14(-1) \\ 18-6p &= -14 \\ -6p &= -32 \\ p &= \frac{-32}{-6} \\ &= \frac{16}{3} \text{ or } 5\frac{1}{3} \end{aligned}$$

4.



You need the coordinates of the midpoints of $\overline{AB}$ and $\overline{AC}$ to find the slope of the line through the midpoints.

$$\begin{aligned}\text{midpoint of } \overline{AB} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{8 - 6}{2}, \frac{2 - 2}{2} \right) \\ &= \left(\frac{2}{2}, \frac{0}{2} \right) \\ &= (1, 0)\end{aligned}$$

$$\begin{aligned}\text{midpoint of } \overline{AC} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{8 + 4}{2}, \frac{2 - 6}{2} \right) \\ &= \left(\frac{12}{2}, \frac{-4}{2} \right) \\ &= (6, -2)\end{aligned}$$

Find the slope of line segment through $(1, 0)$ and $(6, -2)$.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 0}{6 - 1} \\ &= \frac{-2}{5}\end{aligned}$$

$$\begin{aligned}
 \text{slope of } \overline{BC} &= m_{\overline{BC}} = \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-6 - (-2)}{4 - (-6)} \\
 &= \frac{-6 + 2}{4 + 6} \\
 &= \frac{-4}{10} \\
 &= \frac{-2}{5}
 \end{aligned}$$

The slopes are equal. Therefore, the line segments are parallel.

5. Slopes of parallel lines are equal.

$$\begin{aligned}
 \therefore \frac{-3}{7} &= \frac{6}{m} \\
 -3m &= 6 \times 7 \\
 -3m &= 42 \\
 m &= \frac{42}{-3} \\
 &= -14
 \end{aligned}$$

6. Since numbers follow in order, it is easier to find places because you know which direction to go.

Activity 4

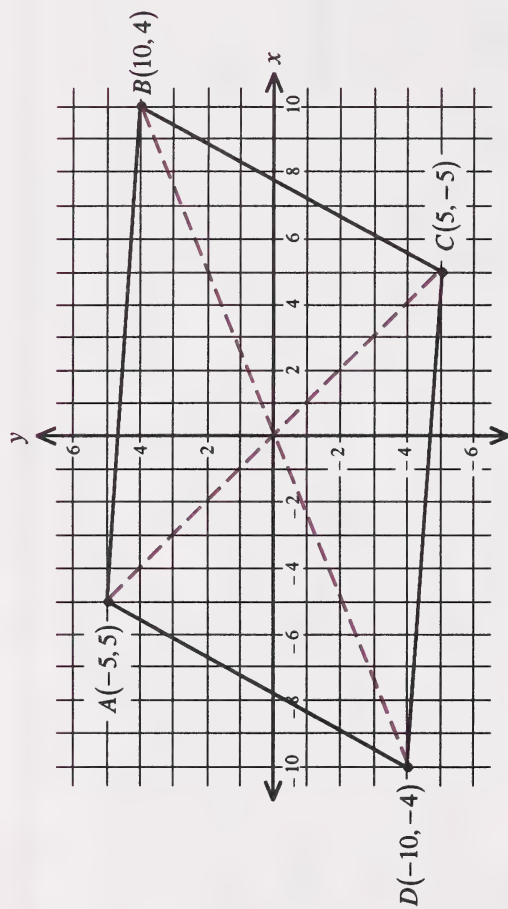
Identify and use the relationship between slopes of perpendicular lines.

1. a. Neither b. Neither
- c. Parallel since $m_1 = m_2$ d. Perpendicular

$$\begin{aligned}
 2. \quad 3k &= \frac{6}{5} \quad \text{or} \quad 3k \times \frac{-5}{6} = -1 \\
 3k \times 5 &= 6 & 3k &= -1 \times \frac{-6}{5} \\
 15k &= 6 & 3k &= \frac{6}{5} \\
 k &= \frac{6}{15} & k &= \frac{6}{5} \times \frac{1}{3} \\
 &= \frac{2}{5} & &= \frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad 4p + 7 &= 5 \\
 4p &= 5 - 7 \\
 4p &= -2 \\
 p &= \frac{-2}{4} \\
 &= \frac{-1}{2}
 \end{aligned}$$

4.



- a. Find the slopes of $\overline{AB}$ and $\overline{DC}$ and the slopes of $\overline{BC}$ and $\overline{AD}$.

$$\begin{aligned}\text{slope of } \overline{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 5}{10 - (-5)} \\ &= \frac{-1}{15}\end{aligned}$$

$$\begin{aligned}\text{slope of } \overline{DC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - (-4)}{5 - (-10)} \\ &= \frac{-1}{15}\end{aligned}$$

Since $m_{\overline{AB}} = m_{\overline{DC}}$, $\overline{AB} \parallel \overline{DC}$.

$$\begin{aligned}\text{slope of } \overline{BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 4}{5 - 10} \\ &= \frac{-9}{-5} \\ &= \frac{9}{5}\end{aligned}$$

Since $m_{\overline{AB}} = m_{\overline{AD}}$, $\overline{BC} \parallel \overline{AD}$.

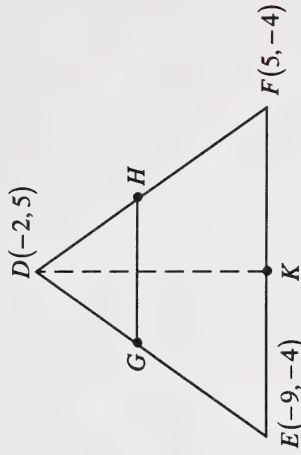
Since $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$, $ABCD$ is a parallelogram.

b. The diagonals are $\overline{AC}$ and $\overline{BD}$.

$$\begin{aligned}\text{slope of } \overline{AC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 5}{5 - (-5)} \\ &= \frac{-10}{10} \\ &= -1\end{aligned}$$

$$\begin{aligned}\text{slope of } \overline{BD} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - 4}{-10 - 10} \\ &= \frac{-8}{-20} \\ &= \frac{2}{5}\end{aligned}$$

5. a. Draw a rough diagram so that you will be able to see the midpoints and the median.



$$\begin{aligned}\text{midpoint of } \overline{DE} = G &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 - 9}{2}, \frac{5 - 4}{2} \right) \\ &= \left(\frac{-11}{2}, \frac{1}{2} \right)\end{aligned}$$

$$\begin{aligned}\text{midpoint of } \overline{DF} = H &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 5}{2}, \frac{5 - 4}{2} \right) \\ &= \left(\frac{3}{2}, \frac{1}{2} \right) \\ &= \left(1\frac{1}{2}, \frac{1}{2} \right)\end{aligned}$$

$$\begin{aligned}\text{slope of } \overline{GH} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{\frac{1}{2} - \frac{1}{2}}{\frac{2}{2} - \left(\frac{-11}{2}\right)} \\ &= 0\end{aligned}$$

The slopes of $\overline{GH}$ and $\overline{EF}$ are equal; therefore, $\overline{GH} \parallel \overline{EF}$.
Since the slope of $\overline{GH}$ and $\overline{EF}$ is zero, $\overline{GH}$ and $\overline{EF}$ are horizontal or parallel to the x -axis.

$$\begin{aligned}\text{b. midpoint of } \overline{EF} = K &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-9 + 5}{2}, \frac{-4 - 4}{2} \right) \\ &= (-2, -4)\end{aligned}$$

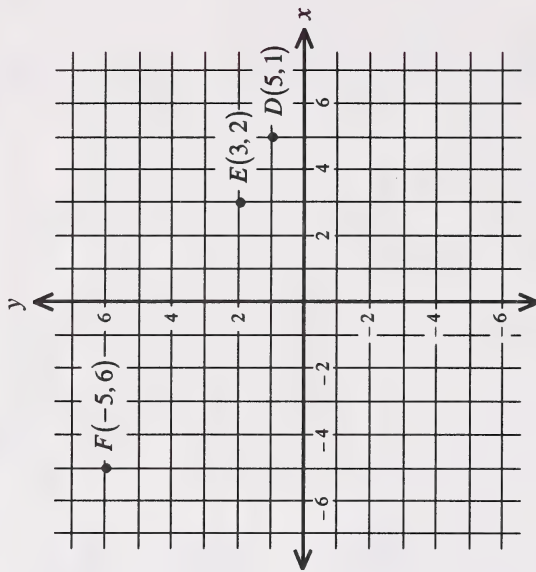
$$\begin{aligned}\text{slope of } \overline{DK} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - 5}{-2 - (-2)} \\ &= \frac{-9}{0} \\ &= \text{undefined}\end{aligned}$$

Since the slope of $\overline{EF} = 0$ and the slope of $\overline{DK}$ is undefined, $\overline{DK} \perp \overline{EF}$. Since the slope of $\overline{DK}$ is undefined, $\overline{DK}$ is vertical or parallel to the y -axis.

Activity 5

Identify collinear points and use slope to determine if points in a plane are collinear.

1. a.



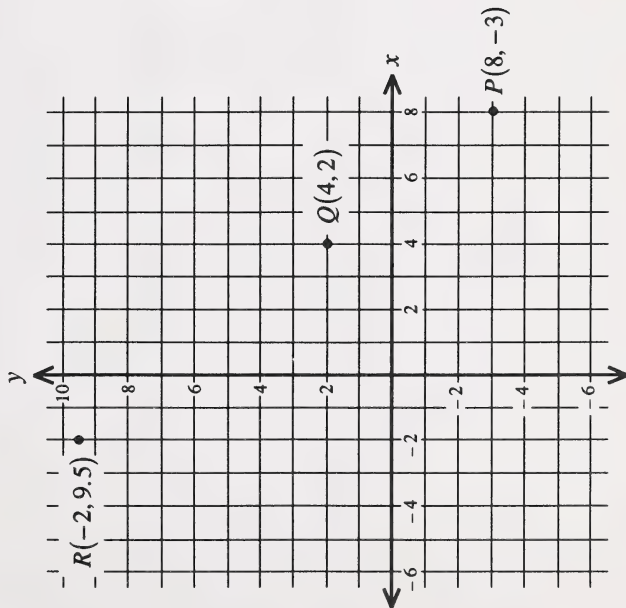
$$\begin{aligned}\text{slope of } \overline{DE} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 1}{3 - 5} \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{slope of } \overline{EF} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 2}{-5 - 3} \\ &= \frac{4}{-8} \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}
 \text{slope of } \overline{DF} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{6 - 1}{-5 - 5} \\
 &= \frac{5}{-10} \\
 &= -\frac{1}{2}
 \end{aligned}$$

Since $m_{\overline{DE}} = m_{\overline{EF}} = m_{\overline{DF}}$, then D , E , and F are collinear.

b.

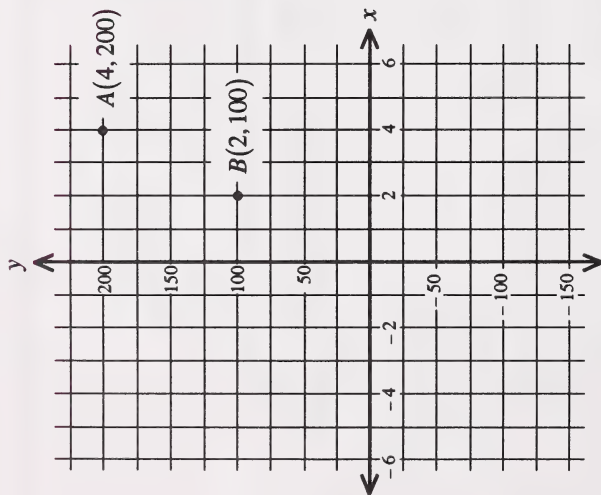


$$\begin{aligned}
 \text{slope of } \overline{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{2 - (-3)}{4 - 8} \\
 &= \frac{5}{-4} \\
 &= -\frac{5}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{slope of } \overline{PR} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{9.5 - (-3)}{-2 - 8} \\
 &= \frac{12.5}{-10} \\
 &= \frac{5}{-4} \\
 &= -\frac{5}{4}
 \end{aligned}$$

Since $m_{\overline{PQ}} = m_{\overline{QR}} = m_{\overline{PR}}$, then P , Q , and R are collinear.

2.



Since A , B , and C lie on the same line, the slope of $\overline{AB}$ is equal to the slope of $\overline{BC}$.

$$50 = \frac{50}{k-2}$$

$$50(k-2) = 50$$

$$50k - 100 = 50$$

$$50k = 150$$

$$k = \frac{150}{50}$$

$$= 3$$

3. Use the distance formula to find the length of segments $\overline{RM}$, $\overline{MS}$, and $\overline{RS}$. If the length of $\overline{RS}$ is the sum of the lengths of $\overline{RM}$ and $\overline{MS}$, the points R , M , and S must lie on the same line. Therefore, R , M , and S must be collinear.

4. Find the slopes of the lines from Parksville (P) to Aquacity (A), from Aquacity (A) to Forestview (F), and from Parksville (P) to Forestview (F).

$$\begin{aligned} \text{slope of } \overline{BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{150 - 100}{k - 2} \\ &= \frac{50}{k - 2} \end{aligned}$$

$$\begin{aligned} \text{slope of } \overline{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{100 - 200}{2 - 4} \\ &= \frac{-100}{-2} \\ &= 50 \end{aligned}$$

$$\begin{aligned} m_{\overline{PA}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ m_{\overline{AF}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - 1}{5 - 4} \\ &= \frac{-6}{1} \\ &= -6 \end{aligned}$$

$$\begin{aligned}
 m_{\overline{PF}} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-5 - 7}{5 - 3} \\
 &= \frac{-12}{2} \\
 &= -6
 \end{aligned}$$

Since the slopes of the lines are the same, the coordinates lie on the same line. Therefore, Aquacity is on the flight path.

Extra Help

1. a. In the relationship $m = \frac{\text{rise}}{\text{run}}$, rise represents the vertical height and run represents the horizontal distance.
- b. If $m = \frac{9}{11}$, the vertical height is 9.
- c. If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a line, then $(x_2 - x_1)$ represents the run in the slope formula.
- d. If the slope is negative, the line falls to the right.
- e. The slope of a vertical line is undefined.

- f. The slope of a horizontal line is 0.
- g. If two lines are perpendicular, the slope of one line will be the negative reciprocal of the slope of the other line.

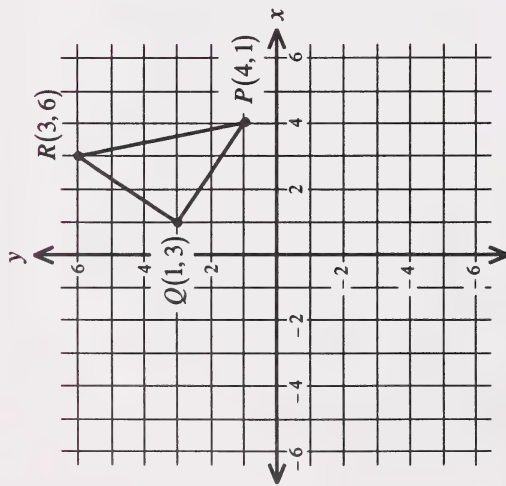
2. The slope of a vertical line is undefined because the run is zero, and division by zero is undefined.

$$\begin{aligned}
 3. \quad \text{rise} &= y_2 - y_1 \quad \text{or} \quad \text{rise} = y_2 - y_1 \\
 &= -11 - 5 \quad \quad \quad = 5 - (-11) \\
 &= -16 \quad \quad \quad = 16
 \end{aligned}$$

$$\begin{aligned}
 4. \quad m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-15 - (-7)}{4 - (-6)} \\
 &= \frac{-15 + 7}{4 + 6} \\
 &= \frac{-8}{10} \\
 &= -\frac{4}{5}
 \end{aligned}$$

The line falls to the right.

5.



$$\begin{aligned} m_{\overline{PQ}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 1}{1 - 4} \\ &= \frac{2}{-3} \\ &= -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} m_{\overline{QR}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 3}{3 - 1} \\ &= \frac{3}{2} \end{aligned}$$

Since $-\frac{2}{3} \times \frac{3}{2} = -1$, $\overline{PQ} \perp \overline{QR}$; thus, $\angle Q$ must be a right angle.

Extensions

1. a. Let $x = 2$.

$$x - 2y = 4$$

$$2 - 2y = 4$$

$$-2y = 4 - 2$$

$$-2y = 2$$

$$y = -1$$

The ordered pair is $(2, -1)$.

Let $x = 0$.

$$x - 2y = 4$$

$$0 - 2y = 4$$

$$y = -2$$

The ordered pair is $(0, -2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - (-1)}{0 - 2} \\ &= \frac{-2 + 1}{-2} \\ &= \frac{-1}{-2} \\ &= \frac{1}{2} \end{aligned}$$

b. Let $x = 1$.

$$2y - 4x = 0$$

$$2y - 4(1) = 0$$

$$2y - 4 = 0$$

$$2y = 4$$

$$y = 2$$

The ordered pair is $(1, 2)$.

Let $x = -1$.

$$2y - 4x = 0$$

$$2y - 4(-1) = 0$$

$$2y + 4 = 0$$

$$2y = -4$$

$$y = -2$$

The ordered pair is $(-1, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

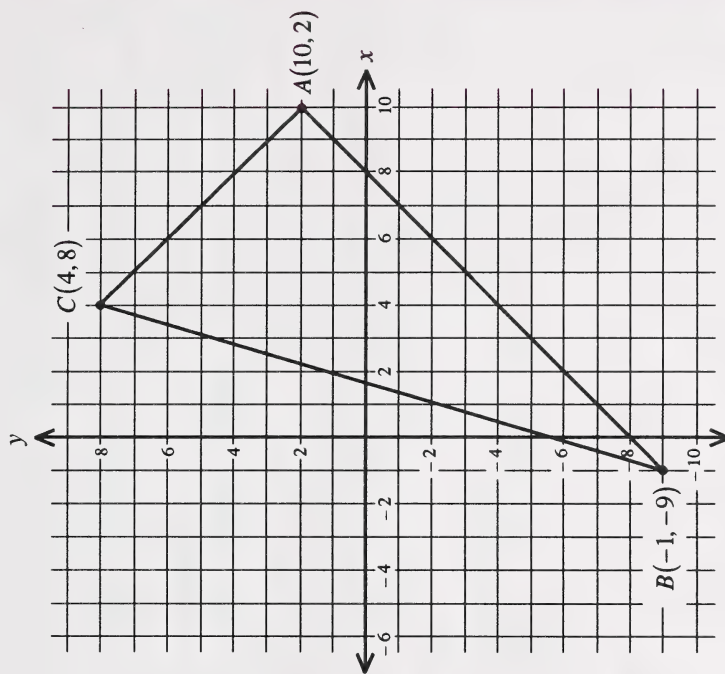
$$= \frac{-2 - 2}{-1 - 1}$$

$$= \frac{-4}{-2}$$

$$= 2$$

Remember: You can choose any value for x .

2. $A(10, 2)$, $B(-1, -9)$, and $C(4, 8)$ are the vertices of $\triangle ABC$.



$$\begin{aligned}
 AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-1 - 10)^2 + (-9 - 2)^2} \\
 &= \sqrt{(-11)^2 + (-11)^2} \\
 &= \sqrt{121 + 121} \\
 &= \sqrt{242}
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[4 - (-1)]^2 + [8 - (-9)]^2} \\
 &= \sqrt{(5)^2 + (17)^2} \\
 &= \sqrt{25 + 289} \\
 &= \sqrt{314}
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(10 - 4)^2 + (2 - 8)^2} \\
 &= \sqrt{(6)^2 + (-6)^2} \\
 &= \sqrt{36 + 36} \\
 &= \sqrt{72}
 \end{aligned}$$

$\overline{BC}$ has the longest length.

Use the Pythagorean theorem to check your answers.

LS	RS
$(BC)^2$	$(AB)^2 + (AC)^2$
$(\sqrt{314})^2$	$(\sqrt{242})^2 + (\sqrt{72})^2$
314	242 + 72
314	314
LS	RS

Therefore, $\triangle ABC$ is a right triangle.



Exploring Topic 4

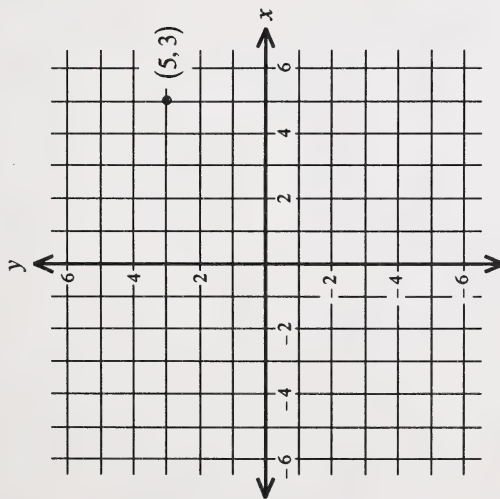
Activity 1

Use terminology and graphing skills related to the graphing of ordered pairs.

- Since both x and y are positive, the ordered pair must be in Quadrant I.
- Since both x and y are negative, the ordered pair must be in Quadrant III.
- This point is not in a quadrant because it lies on the y -axis.

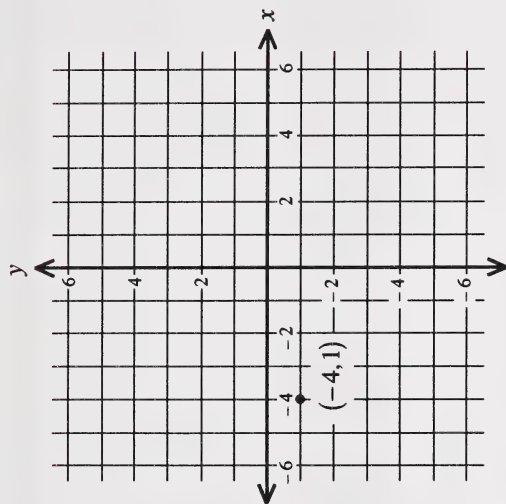
2. The x -coordinate is positive and the y -coordinate is negative.

3.



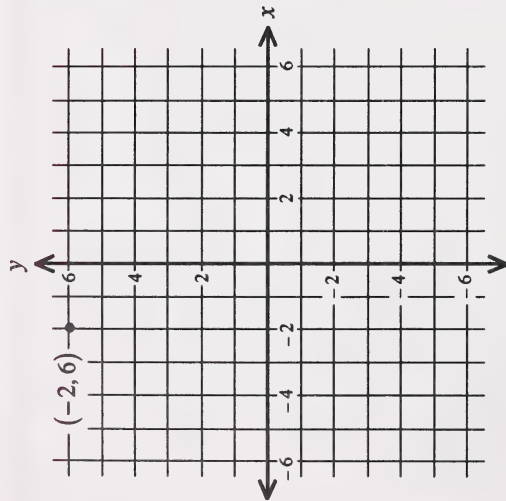
This point is in Quadrant I.

4.



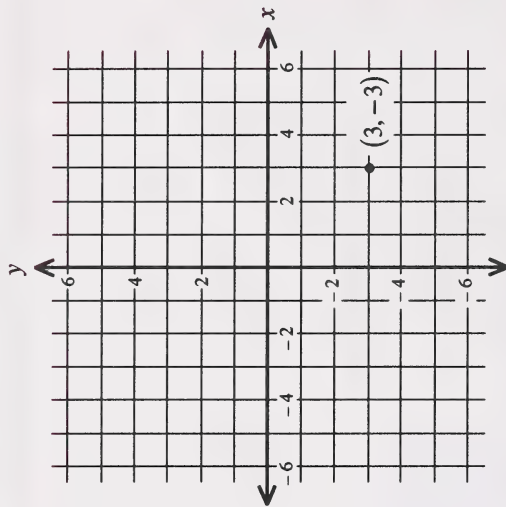
This point is in Quadrant III.

5.



This point is in Quadrant II.

6.



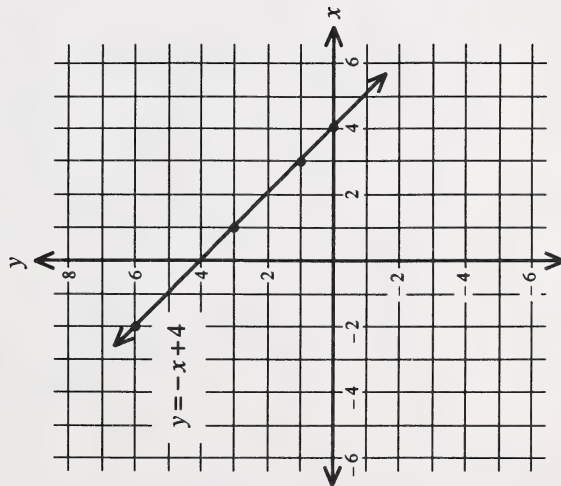
This point is in Quadrant IV.

Activity 2

Develop a table of values, and sketch the graph of a linear equation.

1. a. $y = -x + 4$

x	y
3	1
-2	6
1	3
4	0



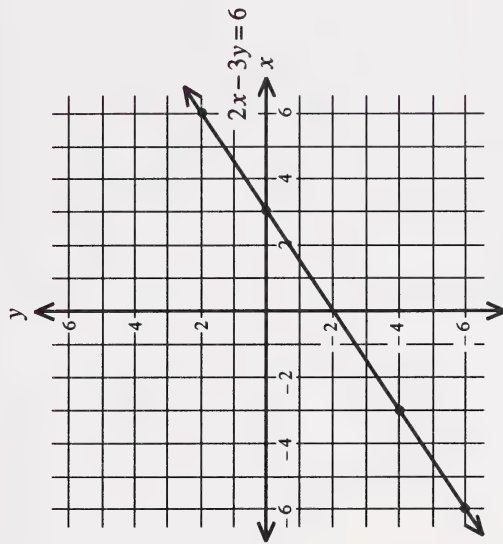
b. $2x - 3y = 6$

$$-3y = -2x + 6$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$$

$$y = \frac{2}{3}x - 2$$

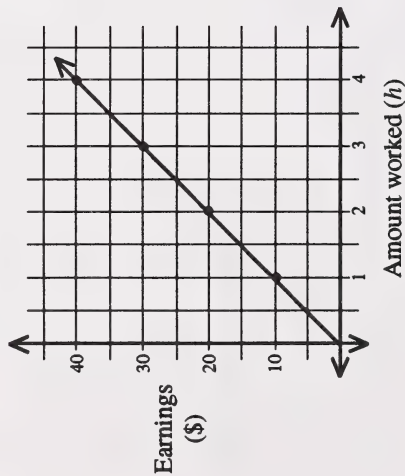
x	y
-6	-6
-3	-4
3	0
6	2



2. a. $y = 10x$

b.

x	y
1	10
2	20
3	30
4	40



- c. It is acceptable to join all points because it is possible to work part of an hour and get paid for it.
- d. The graph cannot be extended in both directions because you cannot work less than zero hours.

- e. You could extend the graph, but this is inconvenient. Substitute 100 for y in the equation.

$$100 = 10x$$

$$x = \frac{100}{10}$$

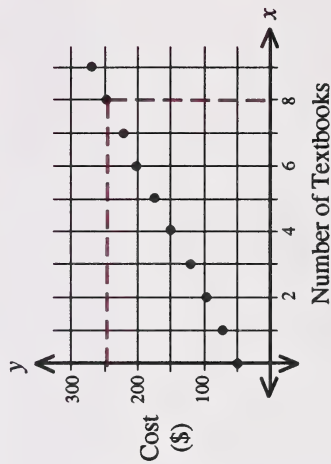
$$= 10$$

Colleen must work 10 h to earn \$100.

3. a. $y = 50 + 25x$

b.

x	y
0	50
1	75
2	100
3	125
4	150

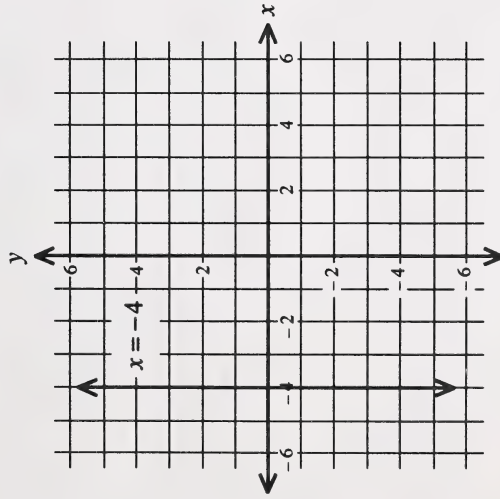


c. The graph starts at \$50 because the minimum cost is \$50.

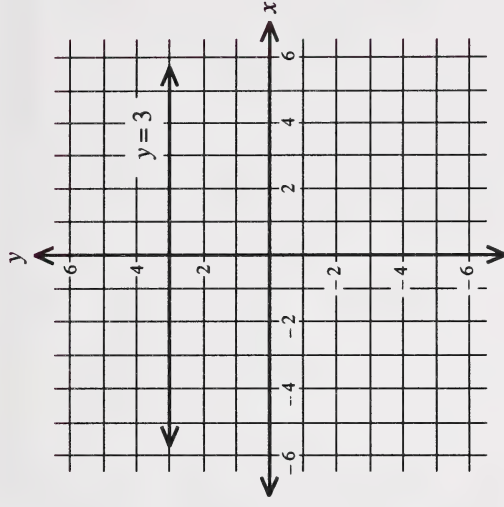
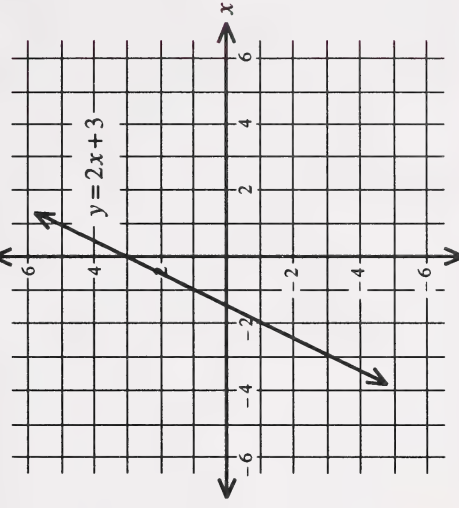
d. The points cannot be connected since you cannot have part of a textbook printed. You cannot print a negative number of textbooks.

e. It will cost \$250 to print eight textbooks.

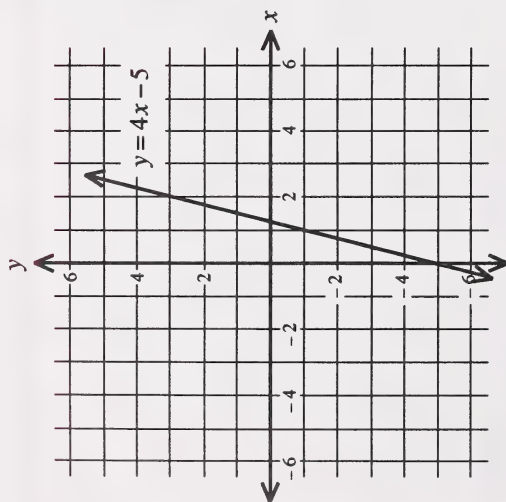
4. a.



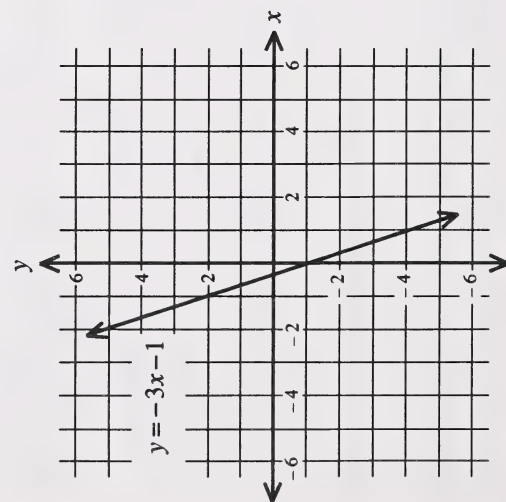
5. a.



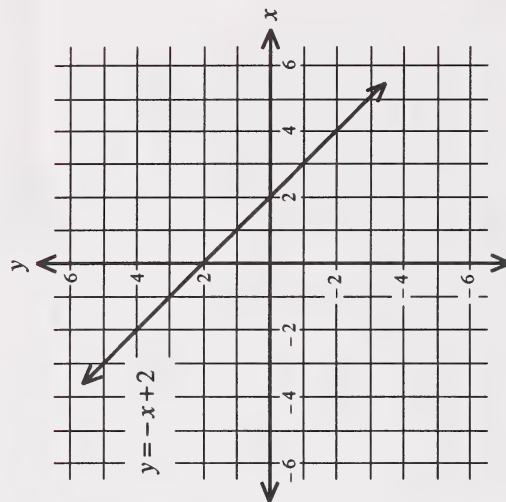
b.



c.

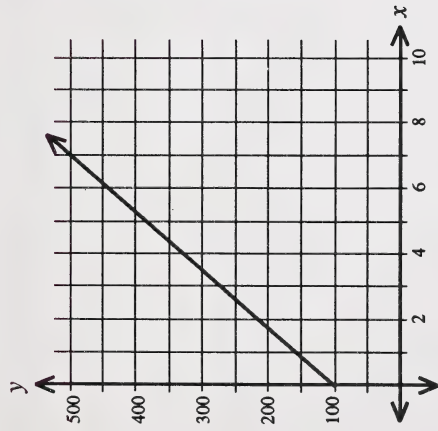


d.



6. The equation for this relation is $y = 100 + 58x$. To graph this equation you need to decide on a range of values for x and y . Since the mechanic cannot work less than zero hours, make x -min and y -min equal to zero. Since 4.75 h is less than 10 h, make x -max equal to ten. To estimate the maximum value for y , use time equal to 5 h. (4.75 rounded to the nearest whole number is 5.) Use cost per hour equal to 60. (58 rounded to the nearest ten is 60.) Five times sixty plus 100 is equal to 400. If you use 400 for the maximum value of y , the cost for 4.75 h should fall within this range. Since each axis has ten markings on it, use a scale of 40 or 50 for the y -axis.

Graph the equation, and use the TRACE and ZOOM function to find the cost.



After using the TRACE and ZOOM functions three times, you should get a cost of \$375.50.

Activity 3

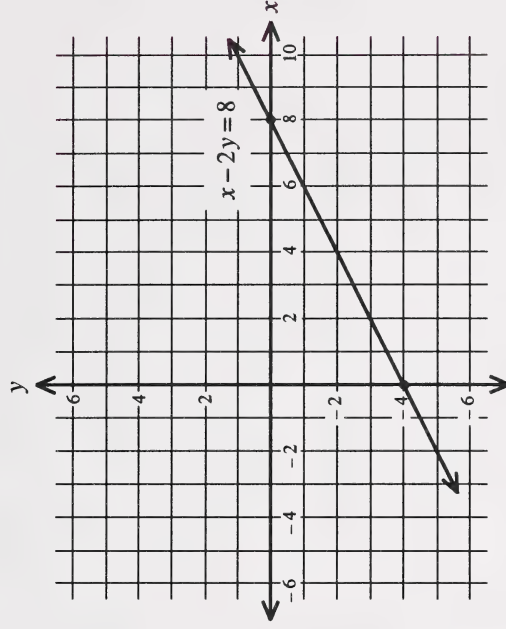
Determine x - and y -intercepts.

1. a. The x -intercept is 4, and the y -intercept is -6 .
- b. There is no x -intercept, and the y -intercept is -10 .
- c. The x -intercept is 0, and the y -intercept is 0.

2. a. When $y = 0$, $x - 2(0) = 8$
 $x = 8$

When $x = 0$, $0 - 2y = 8$
 $-2y = 8$
 $y = -4$

The x -intercept is 8 and the y -intercept is -4 .



b. When $y = 0$, $-3x + 2(0) = 12$

$$-3x = 12$$

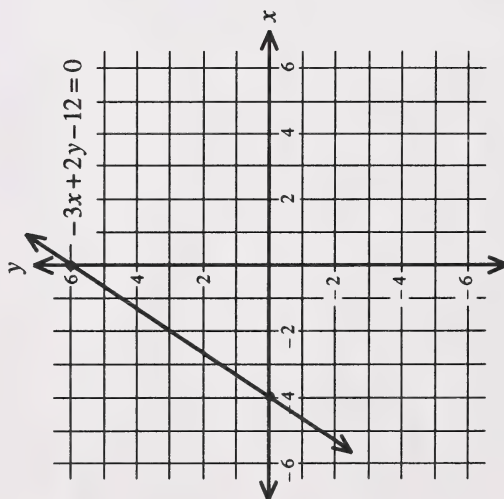
$$x = -4$$

When $x = 0$, $-3(0) + 2y = 12$

$$2y = 12$$

$$y = 6$$

The x-intercept is -4 , and the y-intercept is 6 .



c. When $y = 0$, $0 = \frac{2}{3}x - 2$

$$-\frac{2}{3}x = -2$$

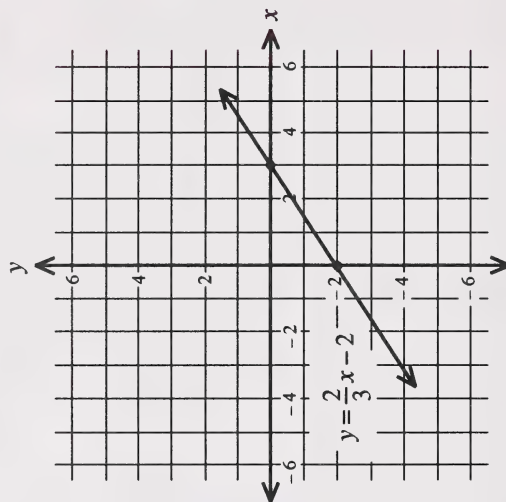
$$-\frac{3}{2} \times \left(-\frac{2}{3}x\right) = -2 \times \left(-\frac{3}{2}\right)$$

$$x = 3$$

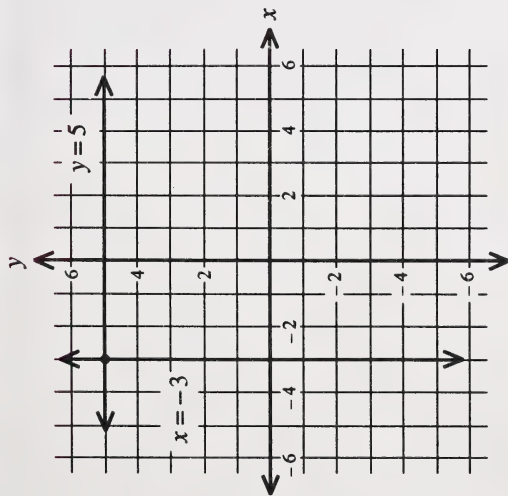
When $x = 0$, $y = 0 - 2$

$$y = -2$$

The x-intercept is 3 , and the y-intercept is -2 .



3.



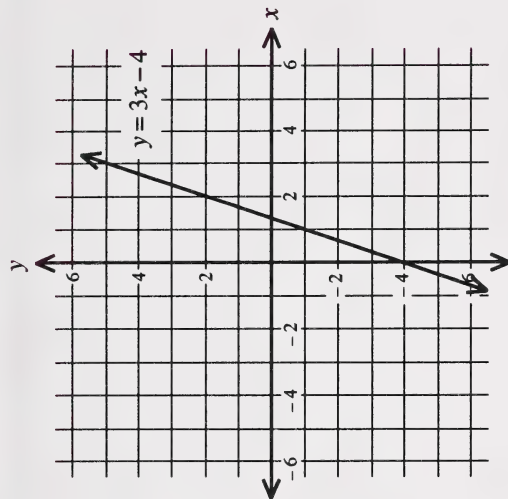
For $x = -3$, the x -intercept is -3 , and there is no y -intercept.

For $y = 5$, there is no x -intercept, and the y -intercept is 5 .

The point of intersection is $(-3, 5)$.

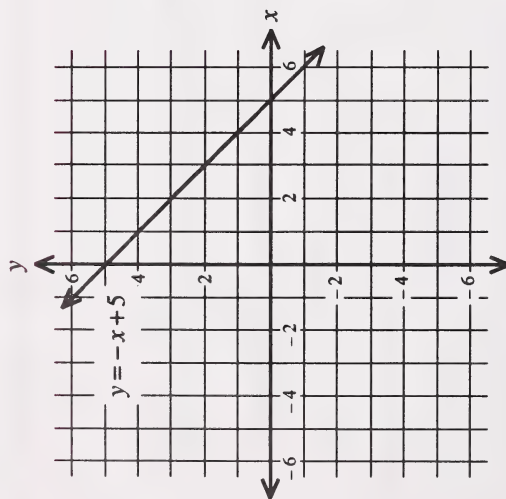
4.
 - a. There is no y -intercept.
 - b. This person is not paid for any work under fifteen minutes.
 - c. The graph is a linear relation. All the points are joined. Thus, the farmhand is paid for any work in excess of fifteen minutes.

5.



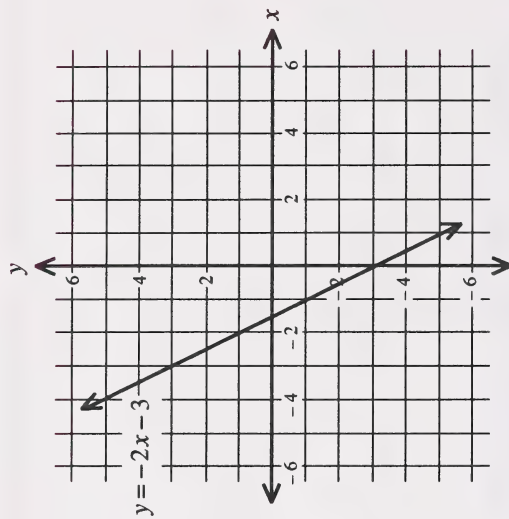
The x -intercept is $\frac{4}{3}$ or $1\frac{1}{3}$, and the y -intercept is -4 .

6.



The x-intercept is 5, and the y-intercept is 5.

7.



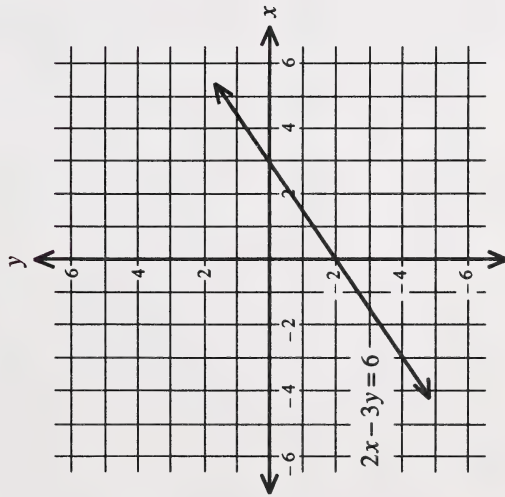
The x-intercept is $-\frac{3}{2}$, and the y-intercept is -3.

8. Change the equation to $y = mx + b$ form.

$$2x - 3y = 6$$

$$-3y = -2x + 6$$

$$y = \left(\frac{2}{3}\right)x - 2$$



The x -intercept is 3, and the y -intercept is -2 .

Extra Help

1. a. Quadrant III b. Quadrant IV
c. Quadrant I d. Quadrant II

2. a. When $y = 0$, $2x - 3(0) = 12$

$$2x = 12$$

$$x = 6$$

$$\text{When } x = 0, 2(0) - 3y = 12$$

$$-3y = 12$$

$$y = -4$$

The x -intercept is 6, and the y -intercept is -4 .

- b. When $y = 0$, $5x - 0 = 0$

$$5x = 0$$

$$x = 0$$

$$\text{When } x = 0, 5(0) - y = 0$$

$$-y = 0$$

$$y = 0$$

The x -intercept is 0, and the y -intercept is 0.

c. When $y = 0$, $-2x + 4(0) = 8$
 $-2x = 8$
 $x = -4$

When $x = 0$, $-2(0) + 4y = 8$
 $4y = 8$
 $y = 2$

The x-intercept is -4 , and the y-intercept is 2 .

d. $2x = 18$
 $x = 9$

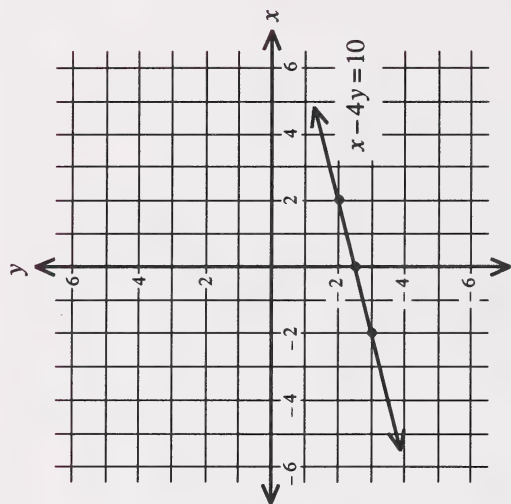
The x-intercept is 9 . There is no y-intercept.

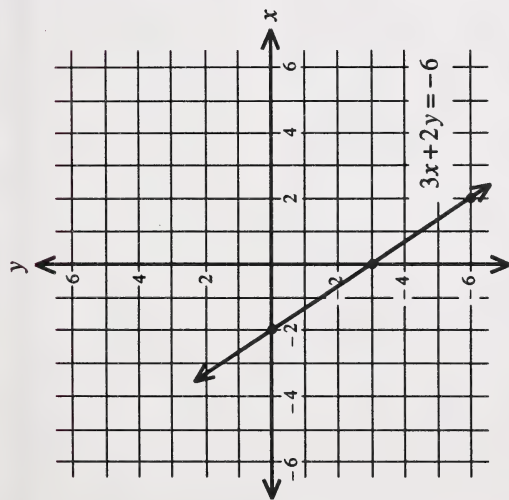
3. a. $x - 4y = 10$

x	y
-2	-3
0	-2.5
2	-2

b. $3x + 2y = -6$

x	y
-2	0
0	-3
2	-6





4. The point is in Quadrant III because both values are negative.

5. The y-coordinate is 15.

6. The independent variable is x . It is easier to give x a value and then calculate the value for y since y is alone on one side of the equation.

7. a. When $y = 0$, $5x - 2(0) = 7$
 $5x = 7$
 $x = 1.4$

When $x = 0$, $5(0) - 2y = 7$
 $-2y = 7$
 $y = -3.5$

The x -intercept is 1.4, and the y -intercept is -3.5 .

b. When $y = 0$, $4x + 16(0) = 0$
 $x = 0$

When $x = 0$, $4(0) + 16y = 0$
 $y = 0$

The x -intercept is 0, and the y -intercept is 0.

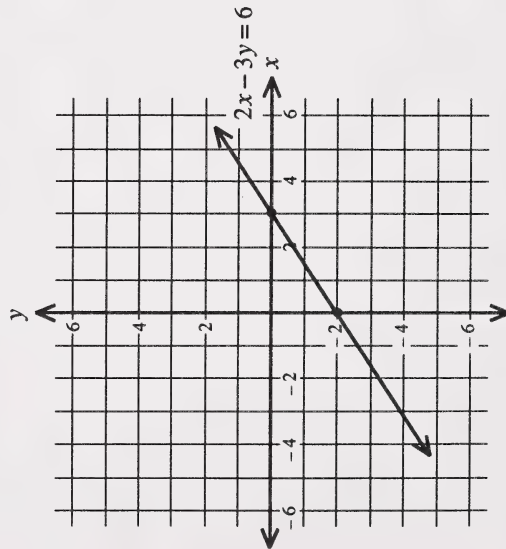
c. $-y = 3$
 $y = -3$

The y -intercept is -3 . There is no x -intercept.

8. a. When $y = 0$, $2x - 3(0) = 6$
 $2x = 6$
 $x = 3$

When $x = 0$, $2(0) - 3y = 6$
 $-3y = 6$
 $y = -2$

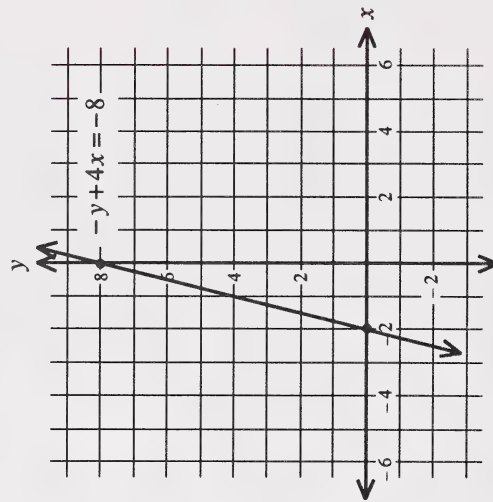
The x -intercept is 3, and the y -intercept is -2 .



- b. When $y = 0$, $0 + 4x = -8$
 $4x = -8$
 $x = -2$

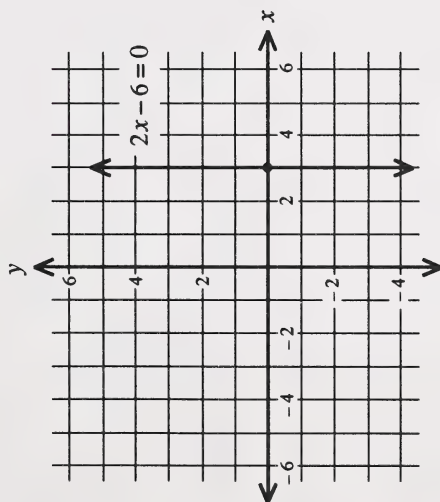
When $x = 0$, $-y + 4(0) = -8$
 $-y = -8$
 $y = 8$

The x -intercept is -2 , and the y -intercept is 8.



c. $2x - 6 = 0$
 $2x = 6$
 $x = 3$

The x -intercept is 3, and there is no y -intercept.



Extensions

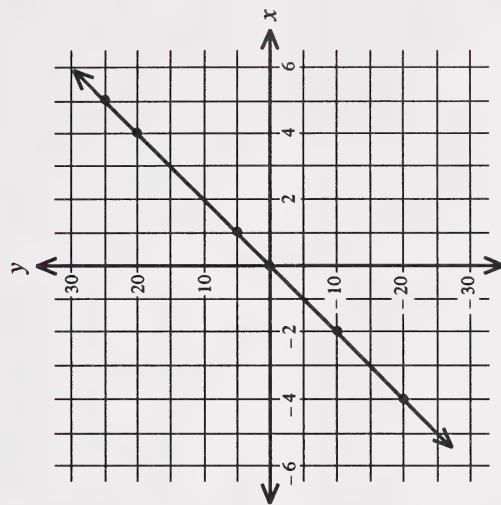
1. a. $20 = m(4)$
 $\frac{20}{4} = m$
 $m = 5$

b.

x	y
-4	-20
-2	-10
0	0
1	5
4	20
5	25

c. $y = 5x$

d.



e. The graph is a straight line.

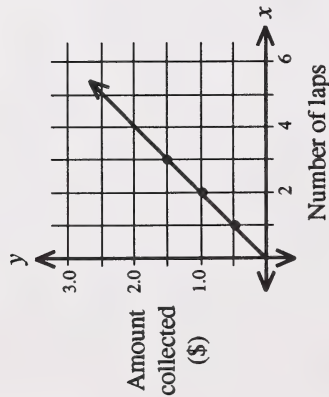
2. a. $50 \times \$0.50 = \$25$

b. $\frac{\$32}{\$0.50/\text{lap}} = 64 \text{ laps}$

- c. Let y be the amount collected.
Let x be the number of laps.
 $\therefore y = 0.50x$

d.

x	y
1	0.50
2	1.00
3	1.50



- e. The graph is a straight line.
f. The graph should not go below the origin because you cannot have negative laps.



Exploring Topic 5

Activity 1

Change the subject in formulas.

1. a. $4x - 3y = 6$

$$-3y = -4x + 6$$

$$\frac{-3y}{-3} = \frac{-4}{-3}x + \frac{6}{-3}$$

$$y = \frac{4}{3}x - 2$$

b. $-x + \frac{1}{2}y - 2 = 0$

$$\frac{1}{2}y = x + 2$$

$$\frac{1}{2}\left(\frac{1}{2}y\right) = 2\left(x + 2\right)(2)$$

$$y = 2x + 4$$

$$c. \quad 3xy - 12 = 0$$

$$3xy = 12$$

$$\frac{3xy}{3x} = \frac{12}{3x}$$

$$y = \frac{4}{x}$$

$$2. \quad a. \quad 2x + y = -2$$

$$2x = -y - 2$$

$$\frac{2x}{2} = \frac{-y}{2} - \frac{2}{2}$$

$$x = -\frac{1}{2}y - 1$$

$$b. \quad \frac{3}{4}x - 2y = 24$$

$$\frac{3}{4}x = 2y + 24$$

$$\frac{4}{3}\left(\frac{3}{4}x\right) = \frac{4}{3}(2y) + \frac{4}{3}(24)$$

$$x = \frac{8}{3}y + 32$$

$$c. \quad 7y - 4x - 28 = 0$$

$$-4x = -7y + 28$$

$$\frac{-4x}{-4} = \frac{-7y}{-4} + \frac{28}{-4}$$

$$x = \frac{7}{4}y - 7$$

$$3. \quad A = \frac{1}{2}bh$$

$$\frac{1}{2}bh = A$$

$$2\left(\frac{1}{2}bh\right) = 2(A)$$

$$bh = 2A$$

$$\frac{bh}{b} = \frac{2A}{b}$$

$$h = \frac{2A}{b}$$

$$4. \quad I = Prt$$

$$Prt = I$$

$$\frac{Prt}{Pt} = \frac{I}{Pt}$$

$$r = \frac{I}{Pt}$$

Activity 2

Write linear equations in $y = mx + b$ form, and identify the significance of m and b .

1. a. The slope is 3, and the y -intercept is 7.

b. The slope is $-\frac{1}{2}$, and the y -intercept is -4 .

- c. The slope is $\frac{4}{5}$, and the y-intercept is -2 .

2. a. $x - 2y = 6$

$$-2y = -x + 6$$

$$\frac{-2y}{-2} = \frac{-x}{-2} + \frac{6}{-2}$$

$$y = \frac{1}{2}x - 3$$

b. $2x + 5y + 15 = 0$

$$5y = -2x - 15$$

$$\frac{5y}{5} = \frac{-2x}{5} - \frac{15}{5}$$

$$y = -\frac{2}{5}x - 3$$

c. $-3x - 4y + 12 = 0$

$$-4y = 3x - 12$$

$$\frac{-4y}{-4} = \frac{3x}{-4} - \frac{12}{-4}$$

$$y = \frac{-3}{4}x - (-3)$$

$$y = -\frac{3}{4}x + 3$$

d. $2x - \frac{1}{2}y = 10$

$$-\frac{1}{2}y = -2x + 10$$

$$-2\left(-\frac{1}{2}y\right) = -2(-2x) - 2(10)$$

$$y = 4x - 20$$

3. a. The ordered pairs are $(4, 0)$ and $(0, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 0}{0 - 4}$$

$$= \frac{2}{-4}$$

$$= -\frac{1}{2}$$

The slope is $-\frac{1}{2}$, and the y-intercept is 2.

- b. The slope is undefined, and there is no y-intercept.

- c. The slope is 0 and the y-intercept is 3.

4. $y = 2x - 1$

When $y = 0$, $0 = 2x - 1$

$$2x = 1$$

$$x = \frac{1}{2}$$

The ordered pair is $(\frac{1}{2}, 0)$.

When $x = 0$, $y = 2(0) - 1$

$$= -1$$

The ordered pair is $(0, -1)$.

$$y = \frac{1}{2}x - 1$$

When $y = 0$, $0 = \frac{1}{2}x - 1$

$$\frac{1}{2}x = 1$$

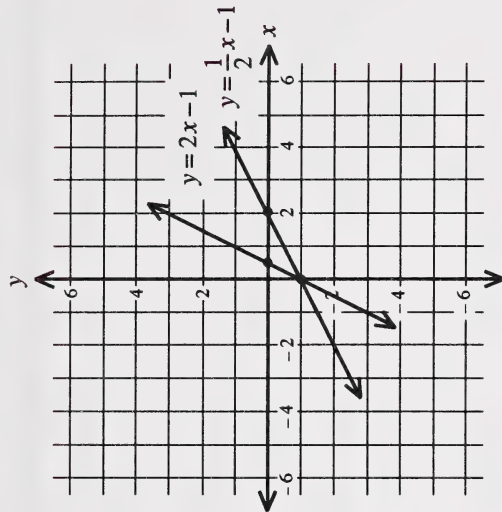
$$x = 2$$

The ordered pair is $(2, 0)$.

When $x = 0$, $y = \frac{1}{2}(0) - 1$

$$= -1$$

The ordered pair is $(0, -1)$.



The slope of $y = 2x - 1$ is steeper than the slope of $y = \frac{1}{2}x - 1$. Therefore, the larger the coefficient of x , the steeper the slope.

5. a. Use a table of values to construct the graphs.

$$y = \frac{2}{3}x$$

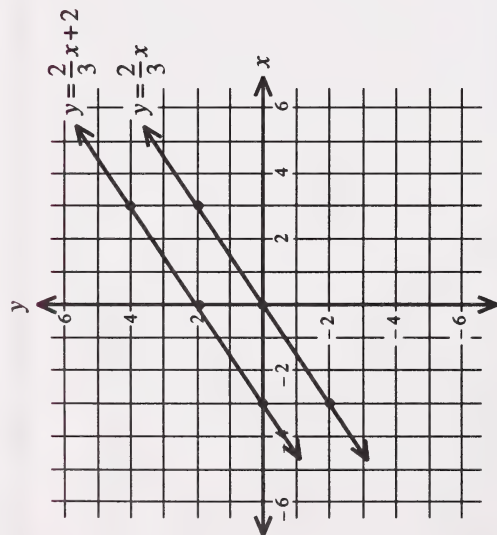
$$y = \frac{2}{3}x + 2$$

x	y
0	0
3	2
-3	-2

x	y
0	2
3	4
-3	0

Activity 3

Determine the slope and the y-intercept of a line from its equation, and draw the graph.



- b. The graphs are parallel. This confirms that the slopes are equal, and that the y-intercepts are distinct. The graph of $y = \frac{2}{3}x + 2$ is the graph of $y = \frac{2}{3}x$ raised two units upwards.

1. a. The slope is $\frac{7}{8}$, and the y-intercept is -5 .
 b. The slope is -2 , and the y-intercept is 2 .
 c. The slope is 4 , and the y-intercept is 0 .
 d. The slope is $-\frac{2}{3}$ and the y-intercept is -3 .
 e. The slope is $-\frac{1}{7}$ and the y-intercept is 0 .
2. a. Substitute $(-3, -8)$ in $y = mx - 5$.

$$y = mx - 5$$

$$-8 = (-3)m - 5$$

$$3m = 8 - 5$$

$$3m = 3$$

$$m = 1$$

$$\therefore y = x - 5$$

- b. Substitute $(-2, \frac{1}{2})$ in $y = mx + 2$.

$$y = mx + 2$$

$$\frac{1}{2} = (-2)m + 2$$

$$2m = 2 - \frac{1}{2}$$

$$2m = 1\frac{1}{2}$$

$$m = \frac{3}{2} \times \frac{1}{2}$$

$$= \frac{3}{4}$$

$$\therefore y = \frac{3}{4}x + 2$$

- c. Substitute $(7, -2)$ in $y = mx - 7$.

$$y = mx - 7$$

$$-2 = 7m - 7$$

$$-7m = -7 + 2$$

$$-7m = -5$$

$$m = \frac{-5}{-7}$$

$$= \frac{5}{7}$$

$$\therefore y = \frac{5}{7}x - 7$$

- d. Substitute $(-4, 2.5)$ in $y = mx + 1$.

$$y = mx + 1$$

$$2.5 = -4m + 1$$

$$4m = 1 - 2.5$$

$$4m = -1.5$$

$$m = \frac{-1.5}{4}$$

$$\left(\text{Rewrite in fraction form as } \frac{-3}{2} = \frac{-3}{2} \times \frac{1}{4} \right)$$

$$= \frac{-3}{2} \times \frac{1}{4}$$

$$= -\frac{3}{8}$$

$$\therefore y = -\frac{3}{8}x + 1$$

3. a. Substitute $(-2, 4)$ in $y = -\frac{1}{2}x + b$.

$$y = -\frac{1}{2}x + b$$

$$4 = -\frac{1}{2}(-2) + b$$

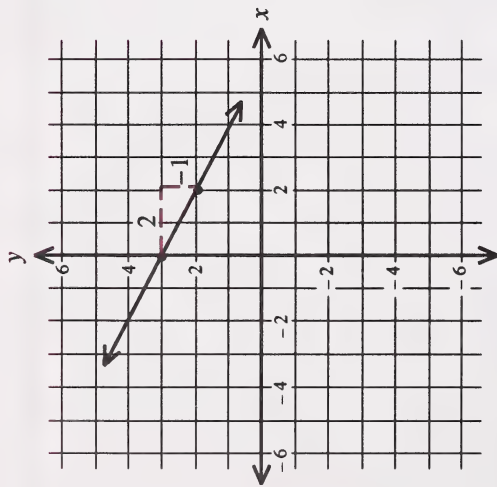
$$4 = 1 + b$$

$$3 = b$$

$$b = 3$$

$$\therefore y = -\frac{1}{2}x + 3$$

To graph the equation, move two units to the right and one unit down from $(0, 3)$. Draw a line through $(0, 3)$ and $(2, 2)$.



- b. Substitute (5, 3) in $y = \frac{2}{5}x + b$.

$$y = \frac{2}{5}x + b$$

$$3 = \frac{2}{5}(5) + b$$

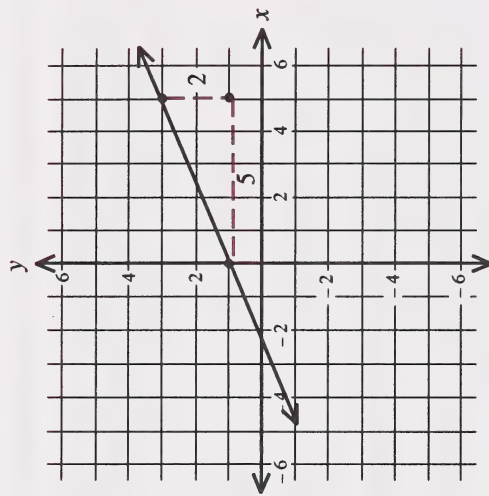
$$3 = 2 + b$$

$$1 = b$$

$$b = 1$$

$$\therefore y = \frac{2}{5}x + 1$$

To graph the equation, move five units to the right and two units up from (0, 1). Draw a line through (0, 1) and (5, 3).



- c. Substitute (-4, 4) in $y = -\frac{3}{2}x + b$.

$$y = -\frac{3}{2}x + b$$

$$4 = -\frac{3}{2}(-4) + b$$

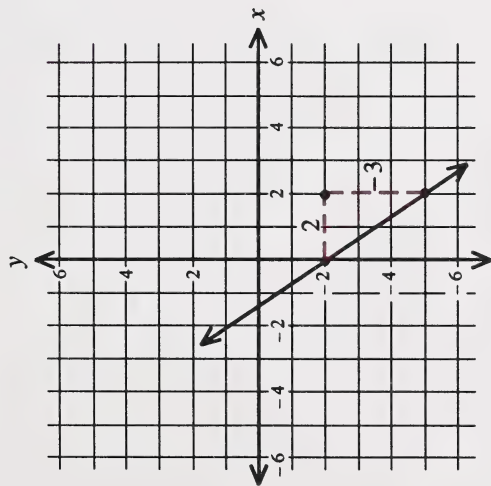
$$4 = 6 + b$$

$$-2 = b$$

$$b = -2$$

$$\therefore y = -\frac{3}{2}x - 2$$

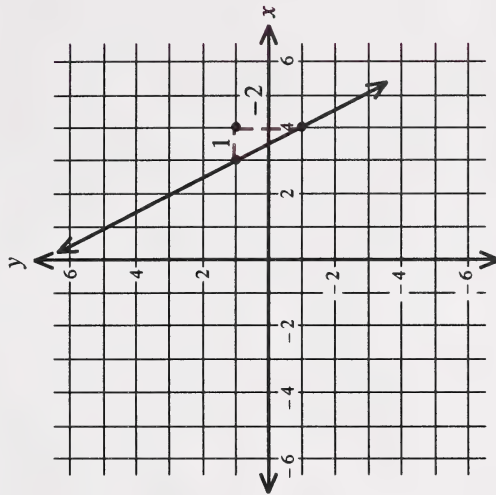
To graph the equation, move two units to the right and three units down from point $(0, -2)$. Draw a line through $(0, -2)$ and $(2, -5)$.



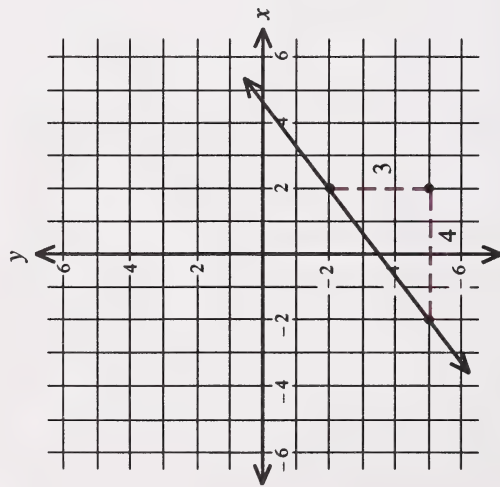
Activity 4

Graph linear equations using slope and another point on the line.

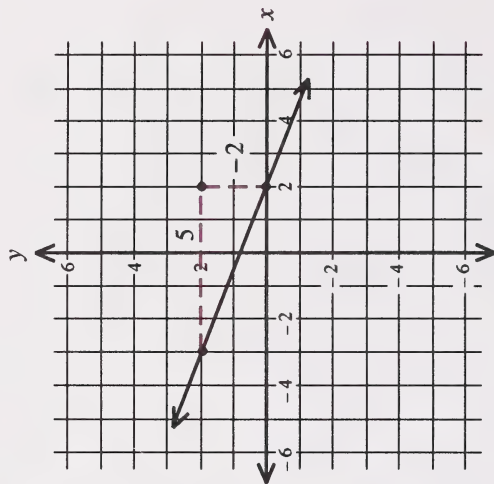
1. a. Starting at point $(3, 1)$, move one unit to the right and two units down. Draw a line through $(3, 1)$ and $(4, -1)$.



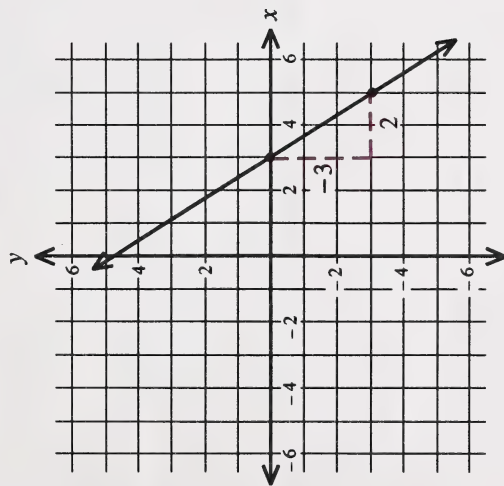
- b. Starting at point $(-2, -5)$, move four units to the right and three units up. Draw a line through $(-2, -5)$ and $(2, -2)$.



- c. Starting at point $(-3, 2)$, move five units to the right and two units down. Draw a line through $(-3, 2)$ and $(2, 0)$.



- d. Starting at point $(3, 0)$, move three units down and two units to the right. Draw a line through $(3, 0)$ and $(5, -3)$.



2. Use the x - and y -intercepts as two points on the line. Then use the slope formula to find m .

- a. The line passes through $(0, 3)$ and $(-2, 0)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - 3}{-2 - 0} \\ &= \frac{-3}{-2} \\ &= \frac{3}{2} \end{aligned}$$

Since the y -intercept is 3, the equation is $y = \frac{3}{2}x + 3$.

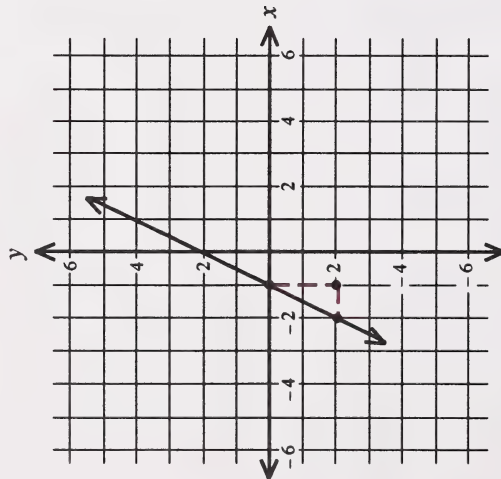
- b. The line passes through $(0, -2)$ and $(-1, 0)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - (-2)}{-1 - 0} \\ &= -2 \end{aligned}$$

Since the y -intercept is -2 , the equation is $y = -2x - 2$.

- c. The line is horizontal. Any line parallel to the x -axis has a zero slope. The y -intercept is -3 . The equation of the line is $y = -3$.

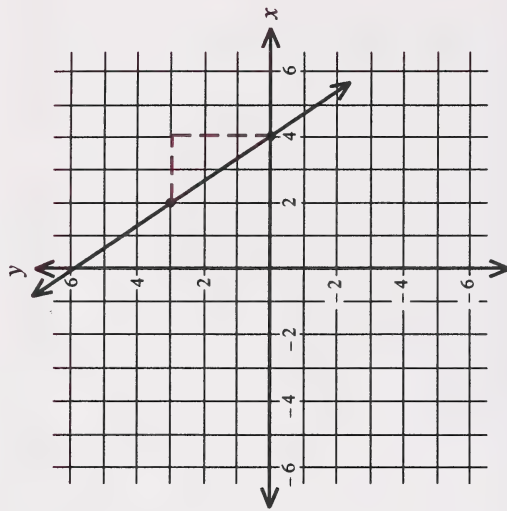
3. a.



From the graph, the x -intercept is -1 , and the y -intercept is 2 .

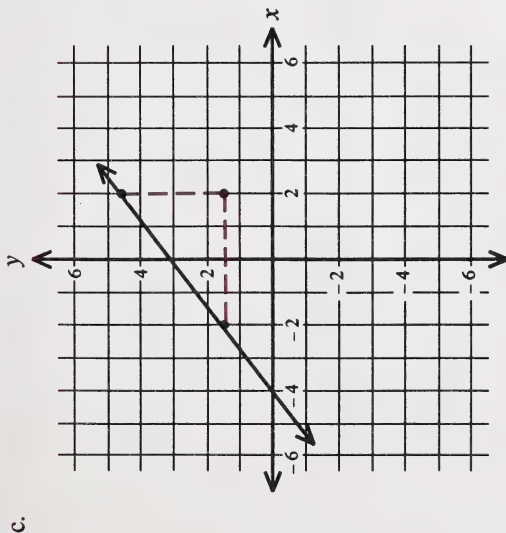
$$\begin{aligned} y &= 2x + b \\ -2 &= 2(-2) + b \\ -2 &= -4 + b \\ b &= 2 \end{aligned}$$

b.



From the graph, the x -intercept is 4 , and the y -intercept is 6 .

$$\begin{aligned} y &= -\frac{3}{2}x + b \\ 3 &= -\frac{3}{2}(2) + b \\ 3 &= -3 + b \\ b &= 6 \end{aligned}$$



4. Choose a convenient point on the line. From this point, draw a horizontal line segment (run) and move away from the point. Then draw a vertical line segment (rise) to meet the graph at a different point. The slope $m = \frac{\text{rise}}{\text{run}}$.

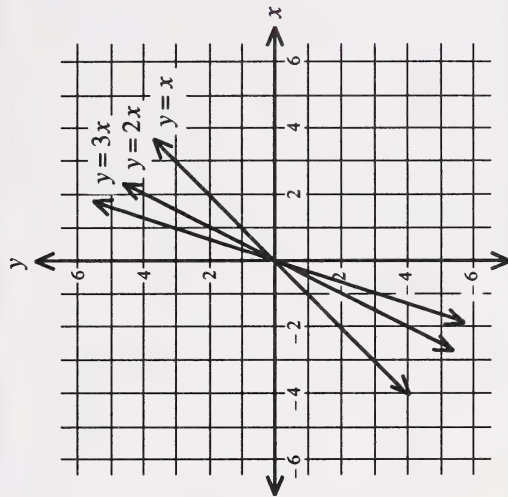
OR

Find the coordinates of the x - and y -intercepts. Then use the slope formula to find the slope.

Activity 5

Use a graphing calculator to study the effects of changing the values of m and b .

- 1.

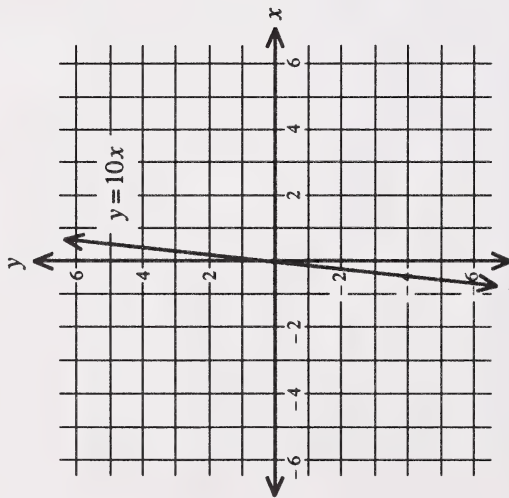


From the graph, the x -intercept is -4 , the y -intercept is 3 .

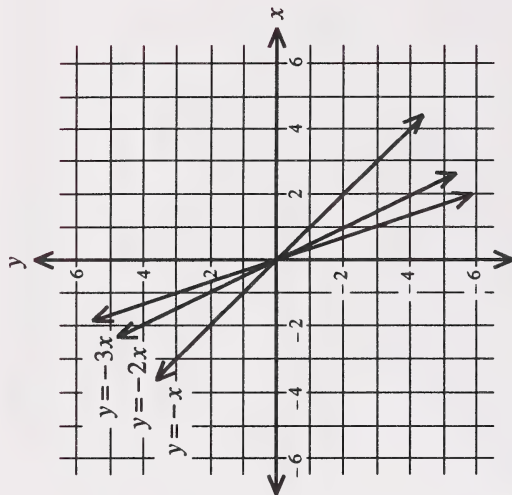
$$\begin{aligned}
 y &= \frac{3}{4}x + b \\
 \frac{1}{2} &= \frac{3}{4}(-2) + b \\
 \frac{3}{2} &= -\frac{3}{2} + b \\
 b &= \frac{3}{2} + \frac{3}{2} \\
 &= \frac{6}{2} \\
 &= 3
 \end{aligned}$$

- a. When m changes from 1 to 2 to 3, the slope rises to the right and becomes steeper. 2.

- b. The slope will rise even steeper to the right.

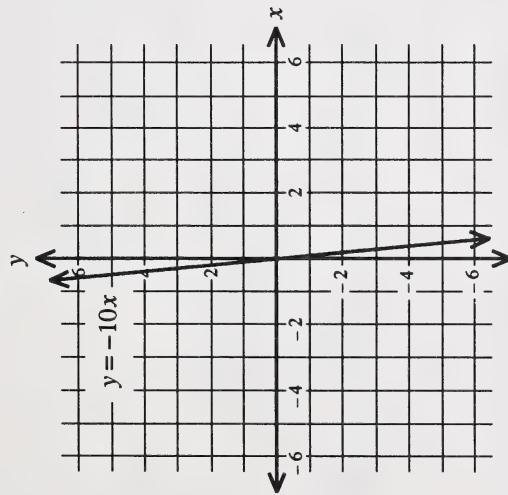


- c. The greater the value of m (when $m > 0$), the steeper the line rises to the right.

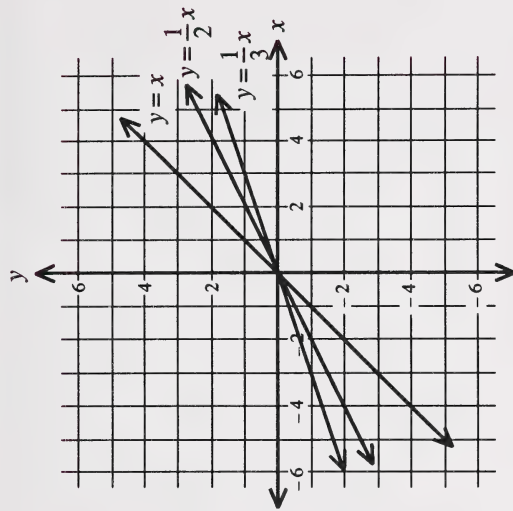


- a. When m changes from -1 to -2 to -3 , the slope falls to the right and becomes steeper.

- b. The slope will fall even steeper to the right.



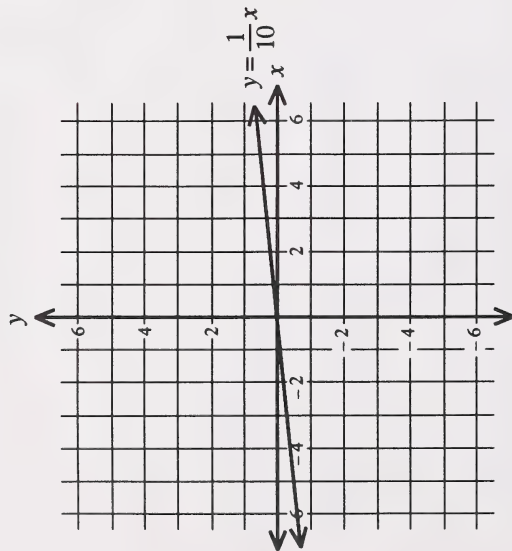
4.



- a. As the size of the fraction decreases, the slope of the line is not as steep.

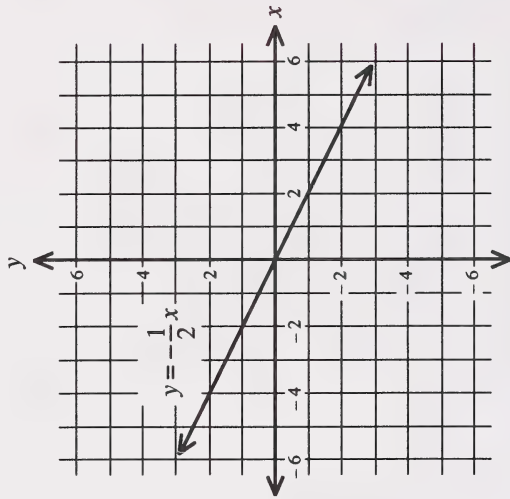
- c. The smaller the value of m (when $m < 0$), the steeper the slope falls to the right.
3. The equations of lines with positive values for m rise to the right, and the equations of lines with negative values for m fall to the right.

- b. The slope of the line will not be as steep as the ones for $m = \frac{1}{2}$ and $m = \frac{1}{3}$.

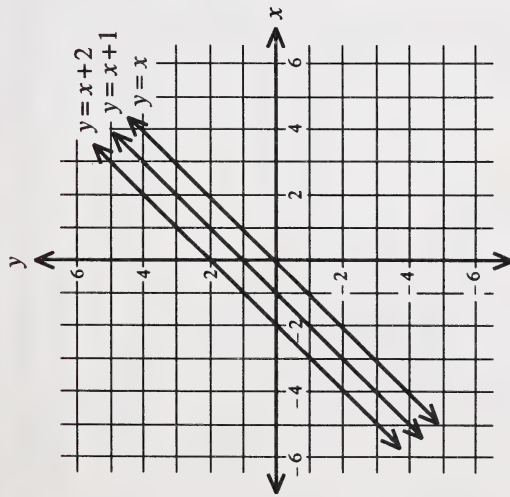


- c. The smaller the fraction, the less steep the slope rises to the right.

- d. The line falls to the right. It will not be as steep as $y = -x$.

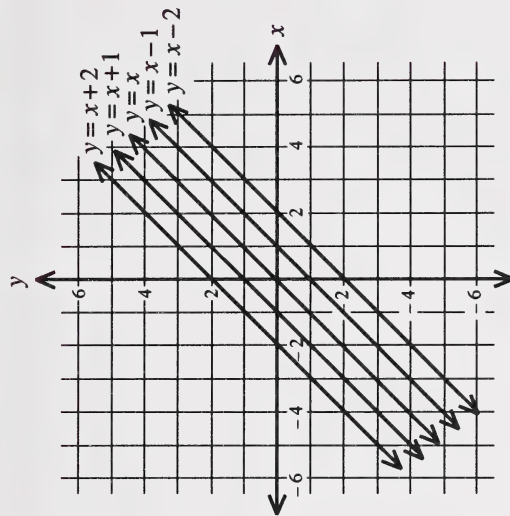


5.



- The line moves up parallel to the line $y = x$. When $b = 0$, the line cuts through $y = 0$; when $b = 1$ the line cuts through $y = 1$; when $b = 2$, the line cuts through $y = 2$.
- When $b = 10$, the line will cut through $y = 10$ and be parallel to the line where $b = 0$.

6.



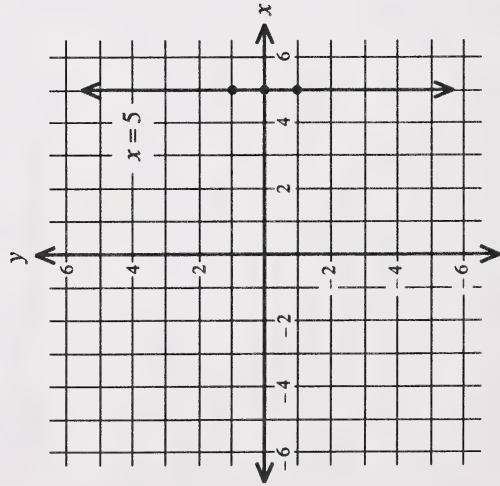
- The lines are parallel. When $b = -1$, the line cuts through $y = -1$; when $b = -2$ the line cuts through $y = -2$.
 - These lines are parallel to the lines in question 5. The negative values of b move the lines below the others.
 - The line will be parallel to the other lines but will cut through $y = -10$.
7. The effects will be the same except the slope of the lines are falling to the right.

Activity 6

Describe and graph equations of lines parallel to the x -axis and lines parallel to the y -axis.

1. a. You can draw a line through $x = 5$ parallel to the y -axis, or you can use a table of values.

x	y
5	-1
5	0
5	1

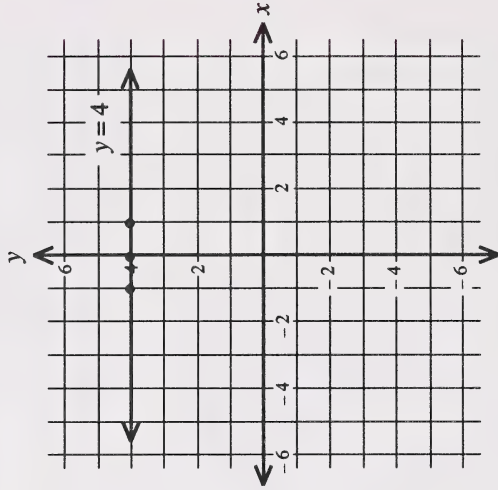


The slope is undefined.

- b. $2y = 8$
 $y = 4$

Draw a line through $y = 4$ parallel to the x -axis, or use a table of values.

x	y
-1	4
0	4
-1	4

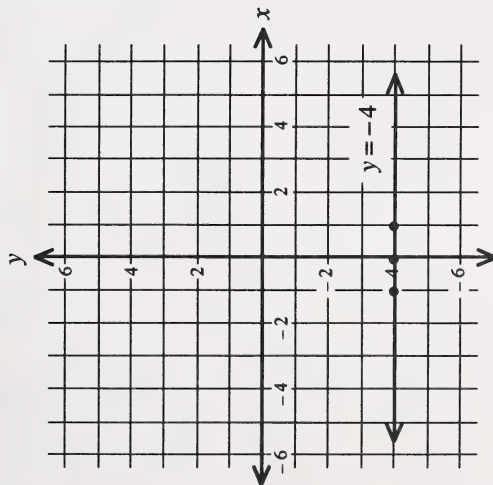


The slope is zero.

c. $3y + 8 = -4$
 $3y = -12$
 $y = -4$

Draw a line through $y = -4$ parallel to the x -axis, or use a table of values.

x	y
-1	-4
0	-4
1	-4

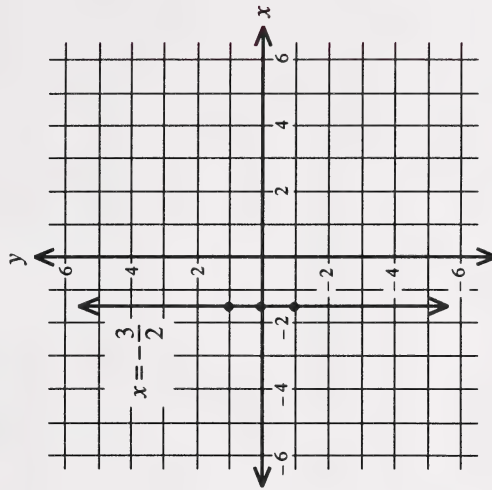


The slope is zero.

d. $2x + 6 = 3$
 $2x = -3$
 $x = -\frac{3}{2}$

x	y
$-\frac{3}{2}$	-1
$-\frac{3}{2}$	0
$-\frac{3}{2}$	1

Draw a line through $x = -\frac{3}{2}$ parallel to the y -axis, or use a table of values.



The slope is undefined.

2. a. The graph is parallel to the y-axis.
 $x = -1$

- b. The graph is parallel to the x-axis.
 $y = -\frac{5}{2}$ or $y = -2\frac{1}{2}$

3. a. Vertical line

- b. Horizontal line

- c. Horizontal line

Extra Help

1. a. $8xy = 36yz$

$$\frac{8xy}{8y} = \frac{36yz}{8y} \quad (\text{Divide both sides by } 8y.)$$

$$x = \frac{9z}{2}$$

- b. $2x + 7 = 4m - 3$

$$2x = 4m - 3 - 7 \quad (\text{Isolate } 2x \text{ by transposing } 7.)$$

$$2x = 4m - 10$$

$$\frac{2x}{2} = \frac{4m - 10}{2} \quad (\text{Divide by } 2 \text{ to isolate } x.)$$

$$x = 2m - 5$$

c. $\frac{3y}{x} = 2$ (Cross multiply.)

$$3y = 2x$$

$$\frac{2x}{2} = \frac{3y}{2} \quad (\text{Change order and divide both sides by } 2.)$$

$$x = \frac{3}{2}y$$

2. a. $4bh = A$ (Change the order of the equation.)

$$\frac{4bh}{4b} = \frac{A}{4b} \quad (\text{Divide both side by } 4b.)$$

$$h = \frac{A}{4b}$$

- b. $64T^2 = h^2$

$$h^2 = 64T^2 \quad (\text{Change the order of the equation.})$$

$$\sqrt{h^2} = \sqrt{64T^2}$$

$$h = \pm 8T$$

c. $2\sqrt{h} = 6mn$

$$(2\sqrt{h})^2 = (6mn)^2 \quad (\text{Square both sides.})$$

$$4h = 36m^2 n^2$$

$$\frac{4h}{4} = \frac{36m^2 n^2}{4}$$

$$h = 9m^2 n^2$$

(Divide both sides by 4.)

3. a. $x - y = 4$

$$-y = -x + 4$$

$$y = x - 4$$

b. $2x + y = 12$

$$y = -2x + 12$$

c. $-2x + 3y = 15$

$$3y = 2x + 15$$

$$y = \frac{2}{3}x + 5$$

4. a. $4y = -12$

$$y = -3$$

b. $5x - 2y = 10$

$$-2y = -5x + 10$$

$$\frac{-2y}{-2} = \frac{-5x}{-2} + \frac{10}{-2}$$

$$y = \frac{5}{2}x - 5$$

c. $4y + x = 4$

$$4y = -x + 4$$

$$y = -\frac{1}{4}x + 1$$

5. a. The slope is -5 , and the y -intercept is 2 .

b. The slope is $\frac{3}{4}$, and the y -intercept is -8 .

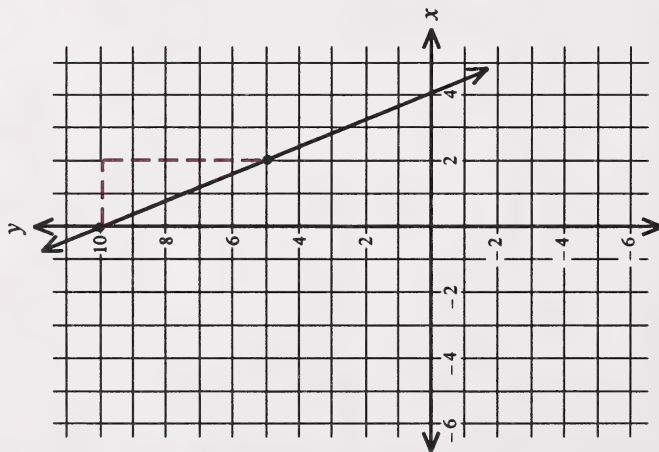
c. The slope is $-\frac{2}{3}$, and the y -intercept is 4 .

6. a. $2y + 5x = 20$

$$2y = -5x + 20$$

$$y = -\frac{5}{2}x + 10$$

The slope is $-\frac{5}{2}$, and the y-intercept is 10.

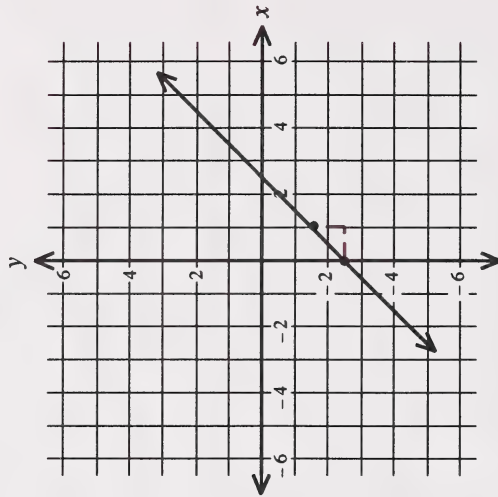


b. $-2x + 2y = -5$

$$2y = 2x - 5$$

$$y = x - \frac{5}{2}$$

The slope is 1, and the y-intercept is $-\frac{5}{2}$.



Extensions

1. a. $d = \frac{1}{2}gt^2$

$$\frac{1}{2}gt^2 = d$$

$$gt^2 = 2d$$

$$t^2 = \frac{2d}{g}$$

$$t = \pm \sqrt{\frac{2d}{g}}$$

c. $E = \frac{WV^2}{2g}$

$$\frac{WV^2}{2g} = E$$

$$WV^2 = 2gE$$

$$V^2 = \frac{2gE}{W}$$

$$V = \pm \sqrt{\frac{2gE}{W}}$$

b. $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$P_1 V_1 = \frac{P_2 V_2 T_1}{T_2}$$

$$V_1 = \frac{P_2 V_2 T_1}{P_1 T_2}$$

$$m = \frac{y - y_1}{x - x_1}$$

$$-\frac{5}{12} = \frac{y - 12}{x - (-2)}$$

$$-\frac{5}{12} = \frac{y - 12}{x + 2}$$

$$-5(x + 2) = 12(y - 12)$$

$$-5x - 10 = 12y - 144$$

$$-5x - 12y - 10 + 144 = 0$$

$$5x + 12y - 134 = 0$$

3. a. $7x + 2y = -6$

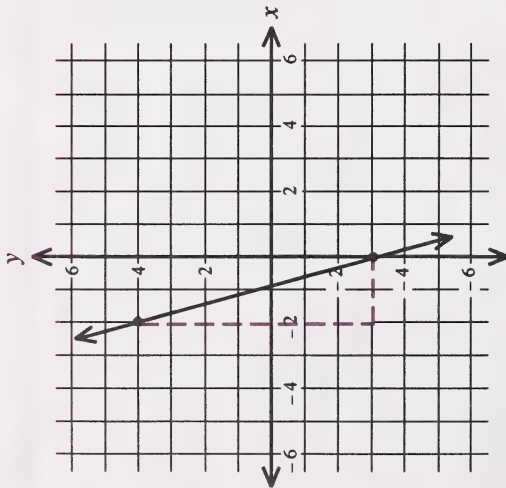
$$2y = -7x - 6$$

$$y = \frac{-7}{2}x - 3$$

b. The slope is $-\frac{7}{2}$.

c. The y-intercept is -3 .

d.



Exploring Topic 6

Activity 1

Rewrite linear equations in $Ax + By + C = 0$ form.

- $4y - 3x = 12$
 $4y - 3x - 12 = 12 - 12$ (Subtract 12 from both sides.)
 $4y - 3x - 12 = 0$ (Write in correct order.)
 $-3x + 4y - 12 = 0$

$A = -3$, $B = 4$, and $C = -12$

You can rewrite $-3x + 4y - 12 = 0$ as $3x - 4y + 12 = 0$. In this case $A = 3$, $B = -4$, and $C = 12$.

- $3x - 5y = -2$
 $3x - 5y + 2 = -2 + 2$
 $3x - 5y + 2 = 0$ (Add 2 to both sides.)

$$9x = 15y - 6$$

$$9x - 15y = 15y - 15y - 6 \quad (\text{Subtract } 15y \text{ from both sides.})$$

$$9x - 15y = -6$$

$$9x - 15y + 6 = -6 + 6$$

(Add 6 to both sides.)

$$9x - 15y + 6 = 0$$

(Divide both sides by 3.)

$$3x - 5y + 2 = 0$$

The values of A , B , and C are the same; therefore, the two equations are equivalent.

3. a. $5y - 3 = -7x$

$$5y - 3 + 7x = -7x + 7x \quad (\text{Add } 7x \text{ to both sides.})$$

$$5y - 3 + 7x = 0$$

(Write in correct order.)

$$7x + 5y - 3 = 0$$

$$A = 7, B = 5, \text{ and } C = -3$$

b. $y = \frac{1}{2}x - 2$

$$y - y = \frac{1}{2}x - 2 - y \quad (\text{Subtract } y \text{ from both sides.})$$

$$0 = \frac{1}{2}x - 2 - y \quad (\text{Write in correct order.})$$

$$\frac{1}{2}x - y - 2 = 0$$

$$A = \frac{1}{2}, B = -1, \text{ and } C = -2$$

c. $8x = -5$

$$8x + 5 = -5 + 5 \quad (\text{Add } 5 \text{ to both sides.})$$

$$8x + 5 = 0$$

This is the same as $8x + 0y + 5 = 0$.

$$A = 8, B = 0, \text{ and } C = 5$$

4. Let $y = 0$.

$$Ax + By + C = 0$$

$$Ax + B(0) + C = 0$$

$$Ax + C = 0$$

$$Ax + C - C = 0 - C$$

$$Ax = -C$$

$$\frac{Ax}{A} = \frac{-C}{A}$$

$$x = \frac{-C}{A}$$

The x -intercept is $x = \frac{-C}{A}$.

Activity 2

Determine the equation of a line given the slope and a point on the line.

1. a. $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{1}{2}[x - (-3)]$$

$$y - 2 = \frac{1}{2}(x + 3)$$

(Multiply both sides by 2.)

$$2(y - 2) = (x + 3)$$

$$(x + 3) = 2(y - 2)$$

$$x + 3 = 2y - 4$$

$$x - 2y + 7 = 0$$

b. $y - y_1 = m(x - x_1)$

$$y - (-1) = 0(x - 5)$$

$$y + 1 = 0(x - 5)$$

$$y + 1 = 0$$

c. $y - y_1 = m(x - x_1)$

$$y - 3 = -3(x - 7)$$

$$y - 3 = -3x + 21$$

$$y - 3 = -3x + 21 + 3x$$

(Add 3x to both sides.)

$$3x + y - 3 = 21$$

(Subtract 21 from both sides.)

$$3x + y - 24 = 0$$

d. $y - y_1 = m(x - x_1)$

$$y - (-2) = -\frac{2}{3}\left(x - \frac{1}{2}\right)$$

(Multiply both sides by 3.)

$$3(y + 2) = -2\left(x - \frac{1}{2}\right)$$

$$3y + 6 = -2x + 1$$

$$2x + 3y + 5 = 0$$

2. The coordinates of a point on the line are $(0, -2)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{3}{4}(x - 0)$$

$$y + 2 = \frac{3}{4}x$$

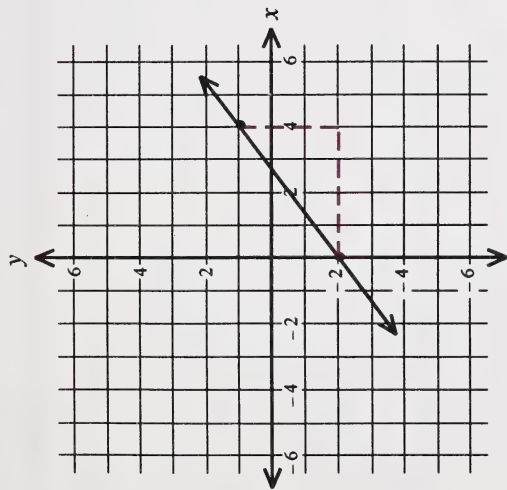
$$4(y + 2) = 3x$$

$$4y + 8 = 3x$$

$$-3x + 4y + 8 = 0$$

(Multiply both sides by -1 .)

$$3x - 4y - 8 = 0$$



3. a. Substitute $(2, 3)$ in $y = -x + 5$.

LS	RS
y	$-x + 5$
3	$-(2) + 5$
3	$-2 + 5$
3	3
LS = RS	

Therefore, $y = -x + 5$ passes through $(2, 3)$.

- b. Substitute $(2, 3)$ in $y = 2x - 3$.

LS	RS
y	$2x - 3$
3	$2(2) - 3$
3	$4 - 3$
3	1
LS $\neq$ RS	

Therefore, $y = 2x - 3$ does not pass through $(2, 3)$.

- c. Substitute $(2, 3)$ in $4x - y = 5$.

LS	RS
$4x - y$	5
$4(2) - 3$	5
$8 - 3$	5
5	5
LS = RS	

Therefore, $4x - y = 5$ passes through $(2, 3)$.

4. a. $(-3, 0)$ are coordinates of the x -intercept.

b. $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{3}{2} [x - (-3)]$$

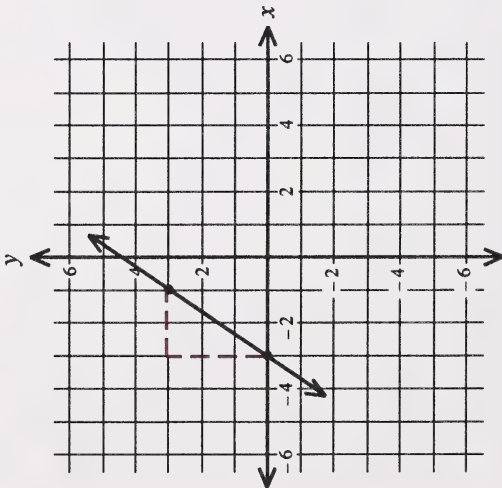
$$y = \frac{3}{2}(x + 3) \quad (\text{Multiply both sides by 2.})$$

$$2y = 3(x + 3)$$

$$2y = 3x + 9$$

$$3x - 2y + 9 = 0$$

c.



5. a. The coordinates of a point on the line are $(0, 4)$.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{-4}{3}(x - 0)$$

$$3(y - 4) = -4x$$

$$3y - 12 = -4x$$

$$4x + 3y - 12 = 0$$

The equation of the line is $4x + 3y - 12 = 0$.

- b. The coordinates of a point on the line are $(-2.5, 0)$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-4}{3} [x - (-2.5)]$$

$$y = \frac{-4}{3}(x + 2.5)$$

$$3y = -4(x + 2.5)$$

$$3y = -4x - 10$$

$$4x + 3y + 10 = 0$$

The equation of the line is $4x + 3y + 10 = 0$.

$$\begin{aligned}
 \text{c.} \quad y - y_1 &= m(x - x_1) \\
 y - (-3) &= \frac{-4}{3} [x - (-4)] \\
 y + 3 &= \frac{-4}{3} (x + 4) \\
 3(y + 3) &= -4(x + 4) \\
 3y + 9 &= -4x - 16 \\
 4x + 3y + 9 + 16 &= 0 \\
 4x + 3y + 25 &= 0
 \end{aligned}$$

The equation of the line is $4x + 3y + 25 = 0$.

6. The amount of snowfall in an area has a bearing on the steepness of the roof. Usually, the greater the snowfall, the steeper the roof.

Activity 3

Determine the equation of a line passing through two given points.

$$\begin{aligned}
 1. \text{ a. } m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-5 - 3}{2 - (-1)} \\
 &= \frac{-8}{2 + 1} \\
 &= \frac{-8}{3}
 \end{aligned}$$

Let $(x_1, y_1) = (-1, 3)$.

$$\begin{aligned}
 y - y_1 &= m(x - x_1) \\
 y - 3 &= \frac{-8}{3} [x - (-1)] \\
 y - 3 &= \frac{-8}{3} (x + 1) \\
 3(y - 3) &= -8(x + 1) \\
 3y - 9 &= -8x - 8 \\
 8x + 3y - 1 &= 0
 \end{aligned}$$

The equation of the line is $8x + 3y - 1 = 0$.

$$\begin{aligned}
 \text{b. } m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{0 - (-4)}{-8 - (-4)} \\
 &= \frac{4}{-8 + 4} \\
 &= \frac{4}{-4} \\
 &= -1
 \end{aligned}$$

$$\text{Let } (x_1, y_1) = (-4, -4).$$

$$\begin{aligned}
 y - y_1 &= m(x - x_1) \\
 y - (-4) &= -1[x - (-4)] \\
 y + 4 &= -1(x + 4) \\
 y + 4 &= -x - 4 \\
 x + y + 8 &= 0
 \end{aligned}$$

The equation of the line is $x + y + 8 = 0$.

$$\begin{aligned}
 \text{c. } m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{3 - 2}{-1 - \frac{1}{2}} \\
 &= \frac{1}{-\frac{3}{2}} \\
 &= -\frac{2}{3}
 \end{aligned}$$

$$\text{Let } (x_1, y_1) = \left(\frac{1}{2}, 2\right).$$

$$\begin{aligned}
 y - y_1 &= m(x - x_1) \\
 y - 2 &= -\frac{2}{3}\left(x - \frac{1}{2}\right) \\
 3(y - 2) &= -2\left(x - \frac{1}{2}\right) \\
 3y - 6 &= -2x + 1 \\
 2x + 3y - 6 - 1 &= 0 \\
 2x + 3y - 7 &= 0
 \end{aligned}$$

The equation of the line is $2x + 3y - 7 = 0$.

$$\begin{aligned} \text{d. } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 2}{8 - 4} \\ &= \frac{-3}{4} \end{aligned}$$

$$\text{Let } (x_1, y_1) = (4, 2).$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{4}(x - 4)$$

$$4(y - 2) = -3(x - 4)$$

$$4y - 8 = -3x + 12$$

$$3x + 4y - 8 - 12 = 0$$

$$3x + 4y - 20 = 0$$

The equation of the line is $3x + 4y - 20 = 0$.

2. The y -intercept is -4 .

The coordinate of this point is $(0, -4)$.

Let $(x_1, y_1) = (0, -4)$ and $(x_2, y_2) = (3, -9)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-9 - (-4)}{3 - 0} \\ &= \frac{-9 + 4}{3} \\ &= \frac{-5}{3} \end{aligned}$$

Let $(x_1, y_1) = (0, -4)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = \frac{-5}{3}(x - 0)$$

$$y + 4 = \frac{-5}{3}(x)$$

$$3(y + 4) = -5(x)$$

$$3y + 12 = -5x$$

$$5x + 3y + 12 = 0$$

The equation of the line is $5x + 3y + 12 = 0$.

3. Let $(x_1, y_1) = (-4, -10)$ and $(x_2, y_2) = (3, -2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - (-10)}{3 - (-4)} \\ &= \frac{-2 + 10}{3 + 4} \\ &= \frac{8}{7} \end{aligned}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-10) &= \frac{8}{7}[x - (-4)] \\ y + 10 &= \frac{8}{7}(x + 4) \\ 7(y + 10) &= 8(x + 4) \\ 7y + 70 &= 8x + 32 \\ -8x + 7y + 38 &= 0 \\ 8x - 7y - 38 &= 0 \end{aligned}$$

(Multiply both sides by -1 .)

The equation of the line is $8x - 7y - 38 = 0$.

4. a. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\begin{aligned} &= \frac{-3 - (-12)}{-7 - (-6)} \\ &= \frac{-3 + 12}{-7 + 6} \\ &= \frac{9}{-1} \\ &= -9 \end{aligned}$$

Let $(x_1, y_1) = (-6, -12)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-12) &= -9[x - (-6)] \\ y + 12 &= -9(x + 6) \\ y + 12 &= -9x - 54 \\ 9x + y + 66 &= 0 \end{aligned}$$

The equation of the line is $9x + y + 66 = 0$.

$$\begin{aligned} \text{b. } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - 3}{1 - 5} \\ &= \frac{-7}{-4} \\ &= \frac{7}{4} \end{aligned}$$

$$\text{Let } (x_1, y_1) = (5, 3).$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= \frac{7}{4}(x - 5) \\ 4(y - 3) &= 7(x - 5) \\ 4y - 12 &= 7x - 35 \\ -7x + 4y + 23 &= 0 \\ 7x - 4y - 23 &= 0 \end{aligned}$$

The equation of the line is $7x - 4y - 23 = 0$.

$$\begin{aligned} \text{5. a. Let } (x_1, y_1) &= (4, -2) \text{ and } (x_2, y_2) = (-1, 6). \\ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - (-2)}{-1 - 4} \\ &= -\frac{8}{5} \end{aligned}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-2) &= -\frac{8}{5}(x - 4) \\ y + 2 &= -\frac{8}{5}(x - 4) \\ 5(y + 2) &= -8(x - 4) \\ 5y + 10 &= -8x + 32 \\ 8x + 5y - 22 &= 0 \end{aligned}$$

The equation of the line is $8x + 5y - 22 = 0$.

- b. Let $(x_1, y_1) = (-5, 2)$ and $(x_2, y_2) = (-3, -3)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 2}{-3 - (-5)} \\ &= \frac{-5}{2} \end{aligned}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-5}{2}[x - (-5)]$$

$$y - 2 = \frac{-5}{2}(x + 5)$$

$$2(y - 2) = -5(x + 5)$$

$$2y - 4 = -5x - 25$$

$$5x + 2y + 21 = 0$$

The equation of the line is $5x + 2y + 21 = 0$.

- c. Let $(x_1, y_1) = (-3, 9)$ and $(x_2, y_2) = (2, -8)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - 9}{2 - (-3)} \\ &= \frac{-17}{5} \end{aligned}$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = \frac{-17}{5}[x - (-3)]$$

$$y - 9 = \frac{-17}{5}(x + 3)$$

$$5(y - 9) = -17(x + 3)$$

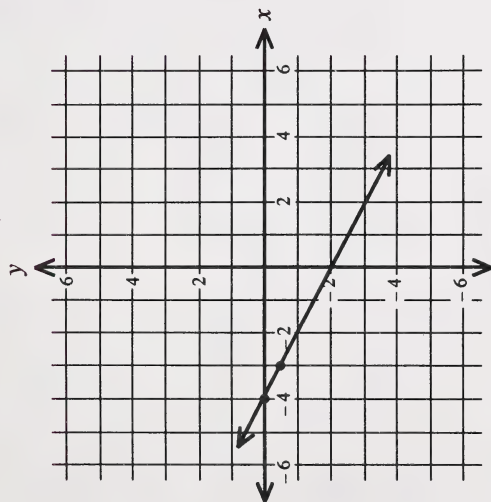
$$5y - 45 = -17x - 51$$

$$17x + 5y + 6 = 0$$

The equation of the line is $17x + 5y + 6 = 0$.

6. a. $(-4, 0)$

b.



c. Let $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (-3, -\frac{1}{2})$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-\frac{1}{2} - 0}{-3 - (-4)} \\ &= \frac{-\frac{1}{2}}{-3 + 4} \\ &= -\frac{1}{2} \end{aligned}$$

The slope of the line is $-\frac{1}{2}$.

d. $y - y_1 = m(x - x_1)$
 $y - 0 = -\frac{1}{2}[x - (-4)]$
 $2(y) = -1(x + 4)$
 $2y = -x - 4$
 $x + 2y + 4 = 0$

The equation of the line is $x + 2y + 4 = 0$.

7. Since P , Q , and R are on the same lines, they are collinear. The slopes of all segments of a line are equal.

$$\begin{aligned} \text{The slope of } \overline{QR} &= \frac{y_1 - y_2}{x_1 - x_2} \\ &= \frac{1 - (-5)}{-4 - 2} \\ &= \frac{6}{-6} \\ &= -1 \end{aligned}$$

Activity 4

Determine the equations of horizontal and vertical lines.

1. $x = k$

2. a. $x = -7$

b. $x = -5$

c. $x = 12$

3. a. $y = -10$

b. $y = -2$

c. $y = 4$

4. A vertical line has a run of zero. Division by zero is undefined.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{rise}}{0} \\ &= \text{undefined} \end{aligned}$$

5. A horizontal line has a rise of zero.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{0}{\text{run}} \\ &= 0 \end{aligned}$$

Activity 5

Determine the equation of a line given one point and the equation of a line parallel or perpendicular to it.

1. $2x - 2y + 3 = 0$

$$-2y = -2x - 3$$

$$y = x + \frac{3}{2} \quad (\text{slope } y\text{-intercept form})$$

$$\therefore m = 1$$

The lines are parallel. Therefore, $m = 1$.

$$y - y_1 = m(x - x_1) \quad (\text{point-slope form})$$

$$y - 3 = 1[x - (-1)]$$

$$y - 3 = (x + 1)$$

$$x - y + 4 = 0$$

The equation of the line is $x - y + 4 = 0$.

$$\begin{aligned}
 2. \quad m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-3 - 1}{-3 - (-2)} \\
 &= \frac{-4}{-3 + 2} \\
 &= \frac{-4}{-1} \\
 &= 4
 \end{aligned}$$

$$y - y_1 = m(x - x_1) \quad (\text{point-slope form})$$

$$y - 2 = 4(x - 4)$$

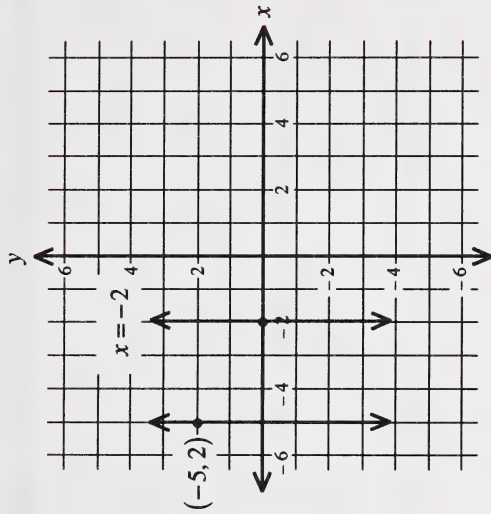
$$y - 2 = 4x - 16$$

$$4x - y - 14 = 0$$

The equation of the line is $4x - y - 14 = 0$.

3. The parallel lines are vertical. The line passes through the point where $x = -5$ and $y = 2$.

The equation of the line is $x = -5$.



4. $4y = x - 8$

$$y = \frac{1}{4}x - 2 \quad (\text{slope } y\text{-intercept form})$$

$$\therefore m = \frac{1}{4}$$

The slope of the line perpendicular to $4y = x - 8$ is -4 since -4 is the negative reciprocal of $\frac{1}{4}$.

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = -4(x - 2)$$

$$y + 4 = -4x + 8$$

$$4x + y - 4 = 0$$

The equation of the line is $4x + y - 4 = 0$.

5. $3x - 2y = 6$

$$-2y = -3x + 6$$

$$y = \frac{3}{2}x - 3 \quad (\text{slope } y\text{-intercept form})$$

$$\therefore m = \frac{3}{2}$$

The slope of the line perpendicular to $3x - 2y = 6$ is $-\frac{2}{3}$, since $-\frac{2}{3}$ is the negative reciprocal of $\frac{3}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{2}{3}[x - (-2)]$$

$$y = -\frac{2}{3}(x + 2)$$

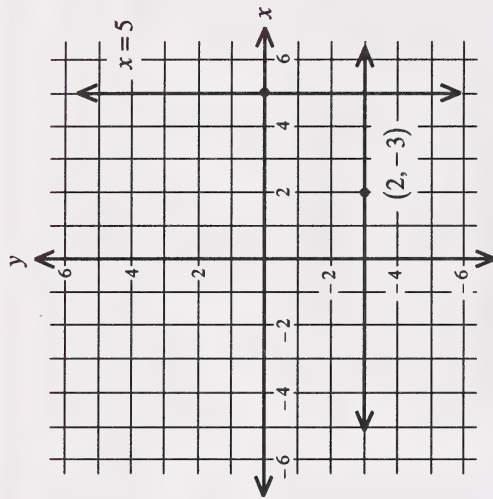
$$3y = -2(x + 2)$$

$$3y = -2x - 4$$

$$2x + 3y + 4 = 0$$

The equation of the line is $2x + 3y + 4 = 0$.

6. The line perpendicular to $x = 5$ is a horizontal line. Therefore, the equation of this line is $y = -3$.



$$\begin{aligned} 7. \quad m_{\overline{AB}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - (-1)}{4 - (-2)} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

The coordinates of the point where the y -intercept is 2 are $(0, 2)$.

Use the point-slope form. The slope of the new line is $\frac{-1 - 3}{\frac{2}{3} - 2}$.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-3}{2}(x - 0)$$

$$2(y - 2) = -3x$$

$$2y - 4 = -3x$$

$$3x + 2y - 4 = 0$$

The equation of the line is $3x + 2y - 4 = 0$.

8. Find the coordinates of point P . P is the midpoint of $\overline{AB}$. Therefore, use the midpoint formula and the coordinates of A and B to find the coordinates of P . Since $\overline{OP}$ is perpendicular to $\overline{AB}$, find the slope of $\overline{OP}$, which is the negative reciprocal of slope $\overline{AB}$. Now use the point-slope form to find the equation of line $\overline{OP}$.

Extra Help

1. a. $-3x + 5y - 6 = 0$ or $3x - 5y + 6 = 0$
 $A = -3$, $B = 5$, and $C = -6$ $A = 3$, $B = -5$, and $C = 6$
- b. $2x - 5y - 2 = 0$
 $A = 2$, $B = -5$, and $C = -2$
- c. $x + 8y - 1 = 0$
 $A = 1$, $B = 8$, and $C = -1$

$$2. \text{ a. } 2x - y + 5 = 0$$

$$\text{b. } x - 7y + 10 = 0$$

$$3. \text{ The slope of } y = -x + 8 \text{ is } -1.$$

$$\text{The slope of } y = x - 2 \text{ is } 1.$$

Therefore, the lines are perpendicular.

$$4. \text{ a. The lines are parallel since their slopes are equal.}$$

$$\text{b. The lines are neither, since their slopes are not related.}$$

$$\text{c. The lines are perpendicular, since their slopes are negative reciprocals.}$$

$$5. \quad y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{4}{5}[x - (-2)]$$

$$y - 4 = -\frac{4}{5}(x + 2)$$

$$5(y - 4) = -4(x + 2)$$

$$5y - 20 = -4x - 8$$

$$4x + 5y - 12 = 0$$

$$\begin{aligned}
 6. \quad m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-5 - (-3)}{-2 - 3} \\
 &= \frac{-2}{-5} \\
 &= \frac{2}{5}
 \end{aligned}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{2}{5}(x - 3)$$

$$y + 3 = \frac{2}{5}(x - 3)$$

$$5(y + 3) = 2(x - 3)$$

$$5y + 15 = 2x - 6$$

$$2x - 5y - 21 = 0$$

Extensions

$$\begin{aligned}
 1. \quad y &= -\frac{4}{5}x + \frac{1}{2} \\
 y + \frac{4}{5}x &= -\frac{4}{5}x + \frac{1}{2} + \frac{4}{5}x \quad (\text{Add } \frac{4}{5}x \text{ to both sides.})
 \end{aligned}$$

$$y + \frac{4}{5}x = \frac{1}{2}$$

$$y + \frac{4}{5}x - \frac{1}{2} = \frac{1}{2} - \frac{1}{2}$$

$$y + \frac{4}{5}x - \frac{1}{2} = 0$$

(Subtract $\frac{1}{2}$ from both sides.)

Now find the L.C.D for $\frac{4}{5}$ and $\frac{1}{2}$.

L.C.D. = 10

Multiply each term by 10.

$$10(y) + 10\left(\frac{4}{5}x\right) + 10\left(-\frac{1}{2}\right) = 0$$

$$10y + 8x - 5 = 0$$

$8x + 10y - 5 = 0$ is in $Ax + By + C = 0$ form.

$A = 8$, $B = 10$, and $C = -5$

2. $-\frac{1}{4}y + \frac{1}{2}x = 3$

$$-\frac{1}{4}y + \frac{1}{2}x - 3 = 3 - 3$$

$$-\frac{1}{4}y + \frac{1}{2}x - 3 = 0$$

Multiply each term by the L.C.D. of $\frac{1}{4}$ and $\frac{1}{2}$.

$$\text{L.C.D.} = 4$$

$$4\left(-\frac{1}{4}y\right) + 4\left(\frac{1}{2}x\right) + 4(-3) = 0$$

$$-y + 2x - 12 = 0$$

$$2x - y - 12 = 0 \text{ is in the form } Ax + By + C = 0.$$

$$A = 2, B = -1, \text{ and } C = -12$$

3. $-\frac{2}{3}x = \frac{5}{6}y - \frac{1}{3}$

$$-\frac{2}{3}x - \frac{5}{6}y = \frac{5}{6}y - \frac{1}{3} - \frac{5}{6}y$$

(Subtract $\frac{5}{6}y$ from both sides.)

$$-\frac{2}{3}x - \frac{5}{6}y = -\frac{1}{3}$$

$$-\frac{2}{3}x - \frac{5}{6}y + \frac{1}{3} = -\frac{1}{3} + \frac{1}{3}$$

(Add $\frac{1}{3}$ to both sides.)

$$-\frac{2}{3}x - \frac{5}{6}y + \frac{1}{3} = 0$$

Find the L.C.D. of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{1}{3}$.

$$\text{L.C.D.} = 6.$$

Multiply each term by 6.

$$6\left(-\frac{2}{3}x\right) + 6\left(-\frac{5}{6}y\right) + 6\left(\frac{1}{3}\right) = 0$$

$$-4x - 5y + 2 = 0$$

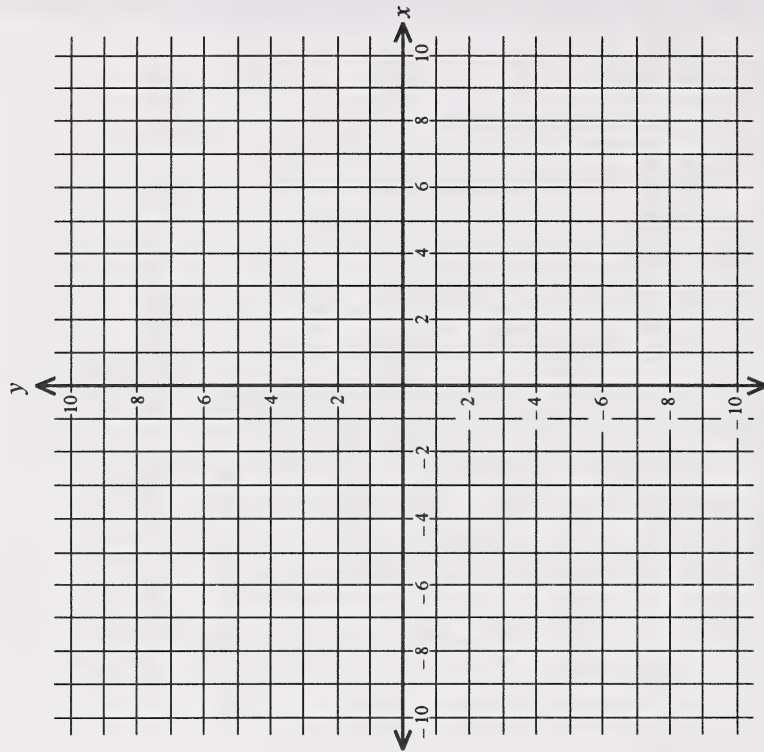
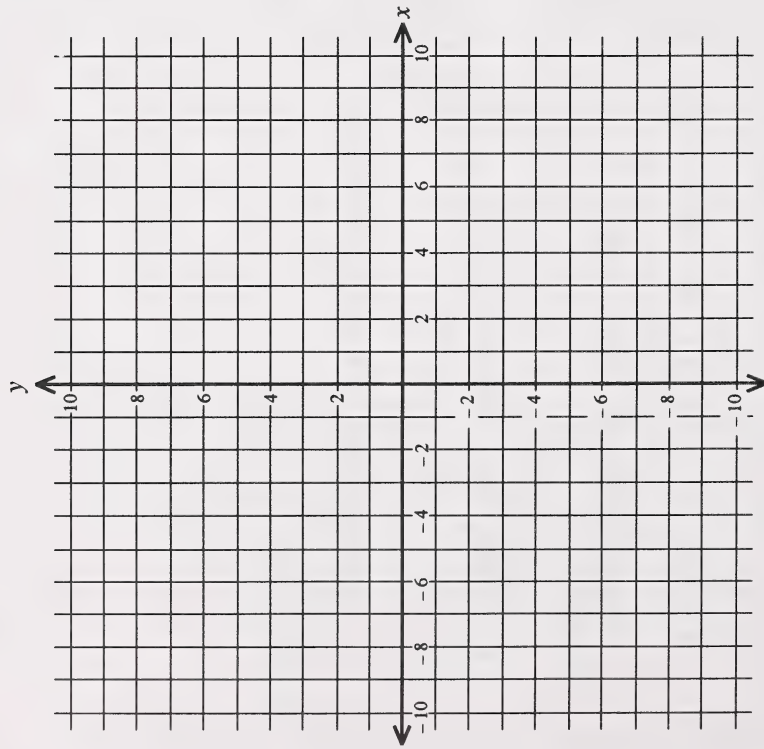
$$A = -4, B = -5, \text{ and } C = 2$$

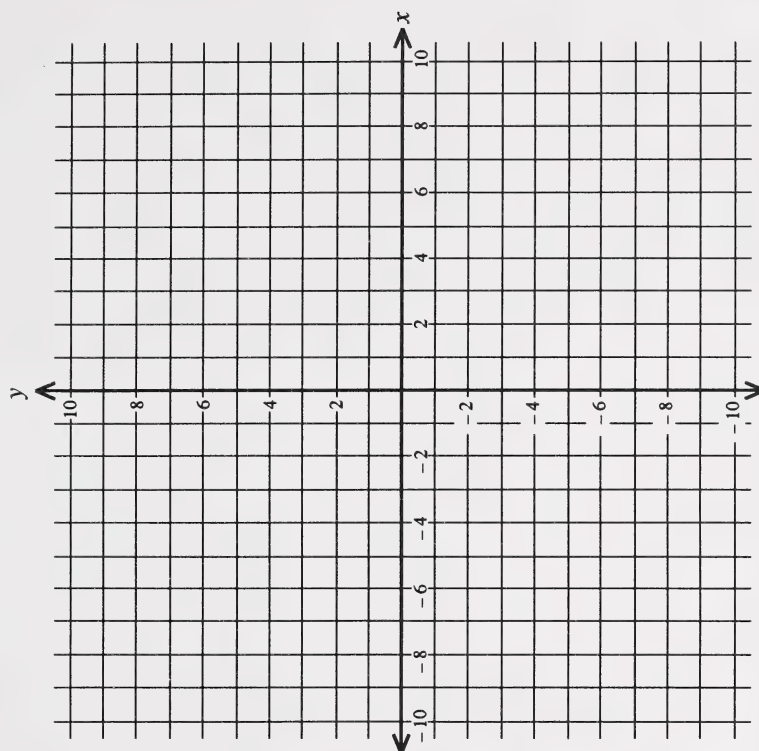
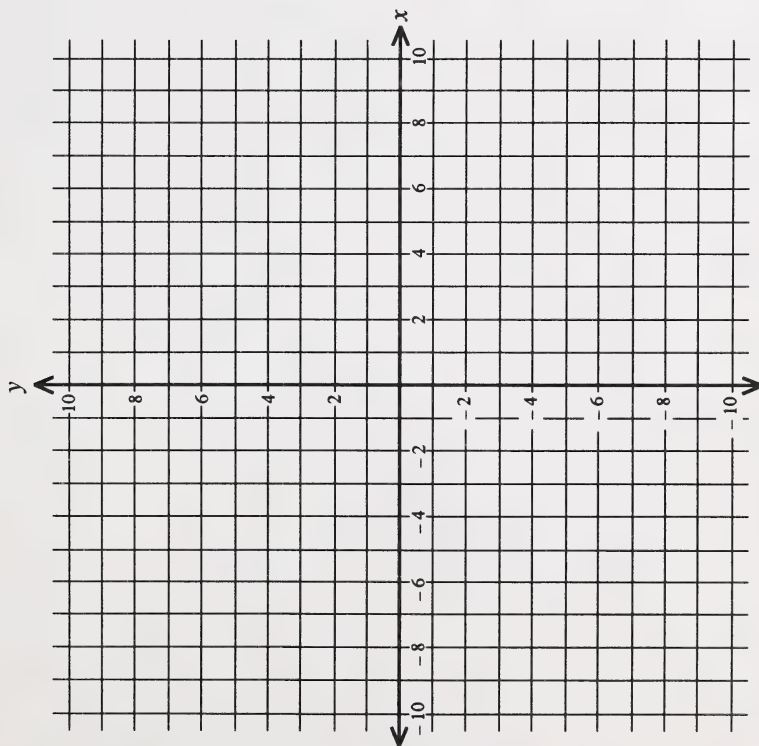
Usually the coefficient of x is made positive. Multiply both sides of the equation by -1 . Therefore, $-4x - 5y + 2 = 0$ becomes $4x + 5y - 2 = 0$. Now $A = 4$, $B = 5$, and $C = -2$.

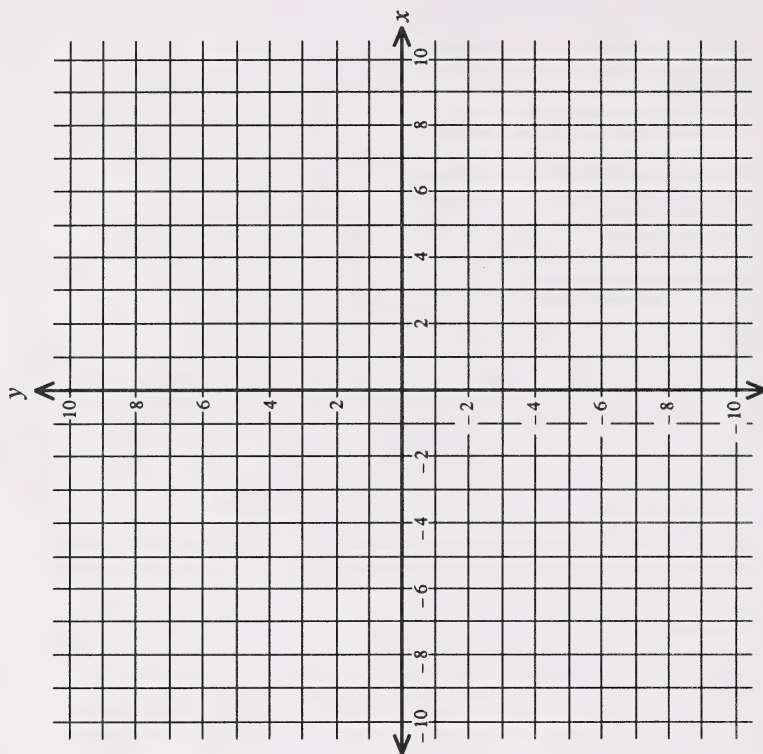
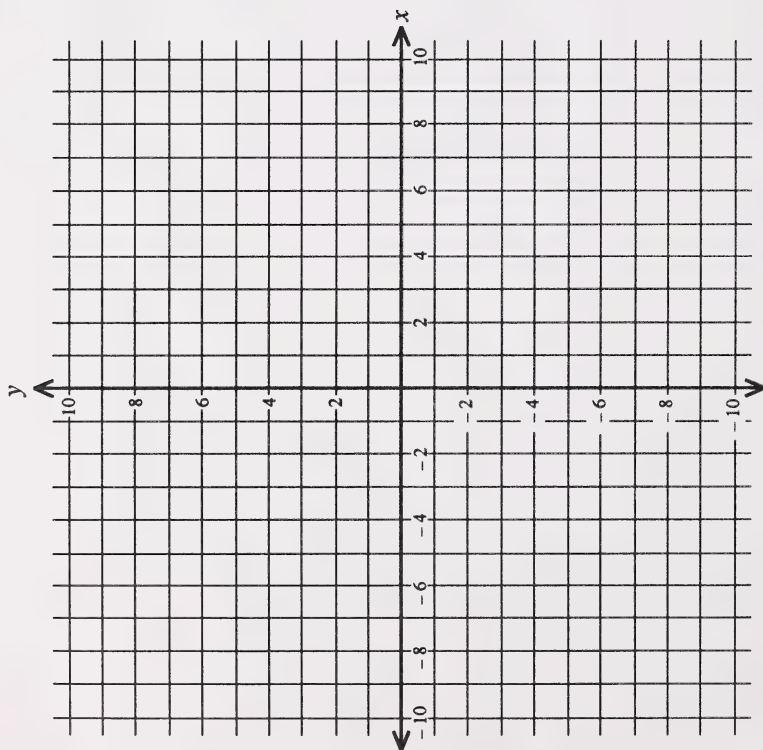


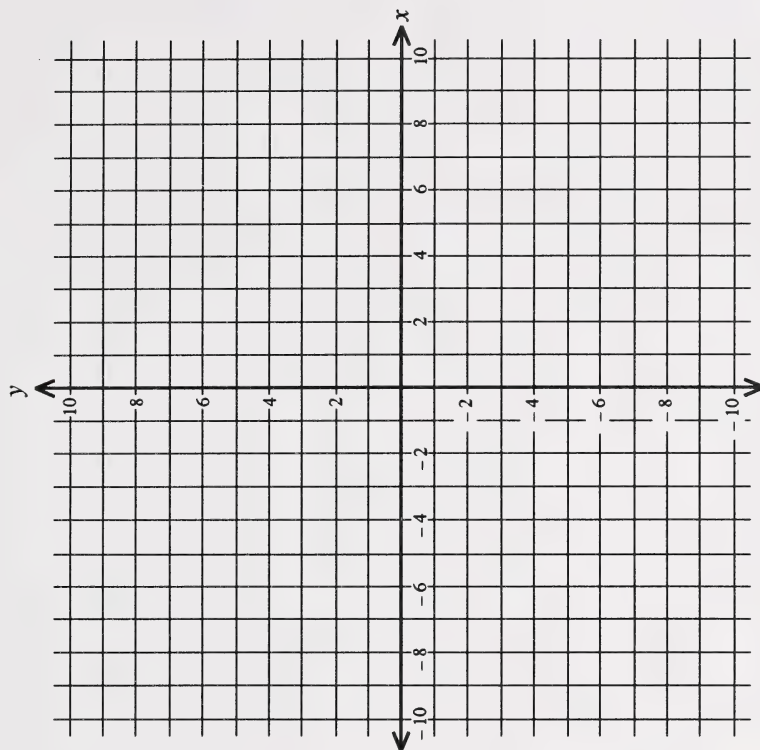
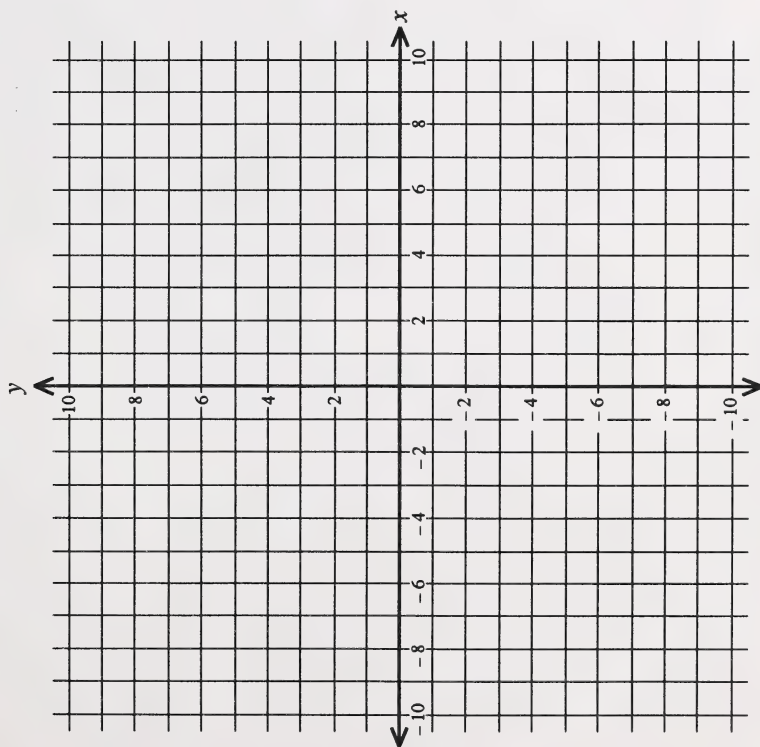
Appendix B Graph Paper

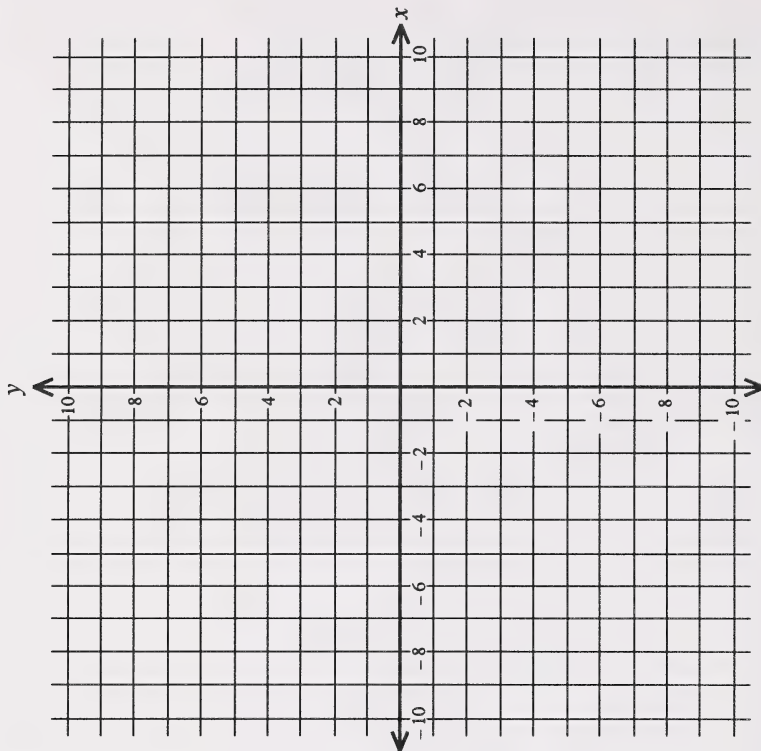
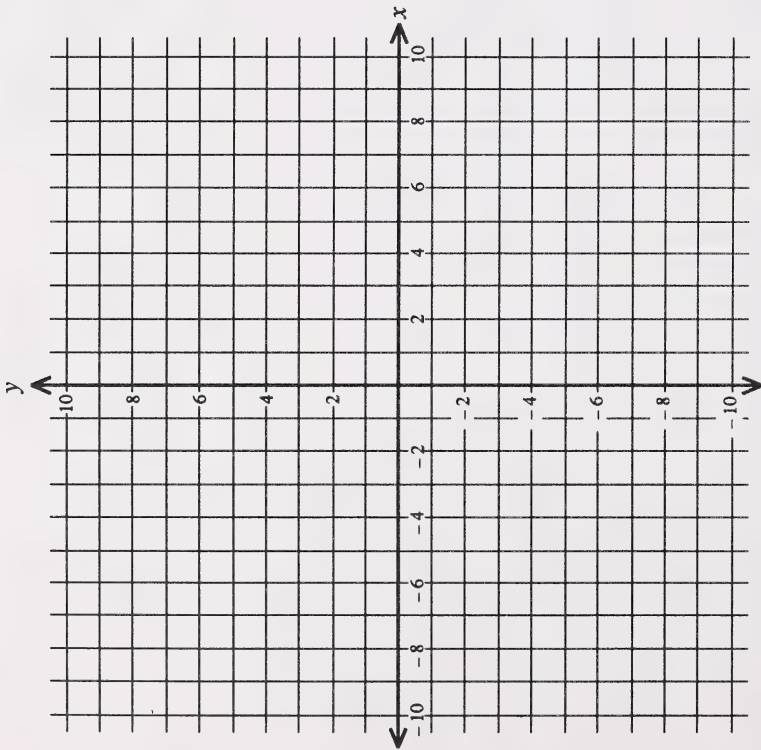
Grids

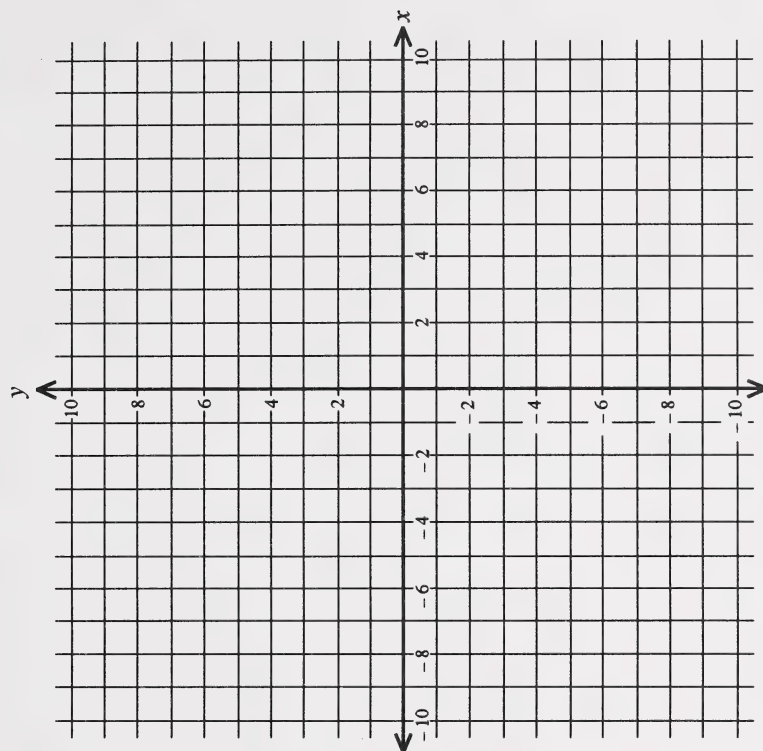
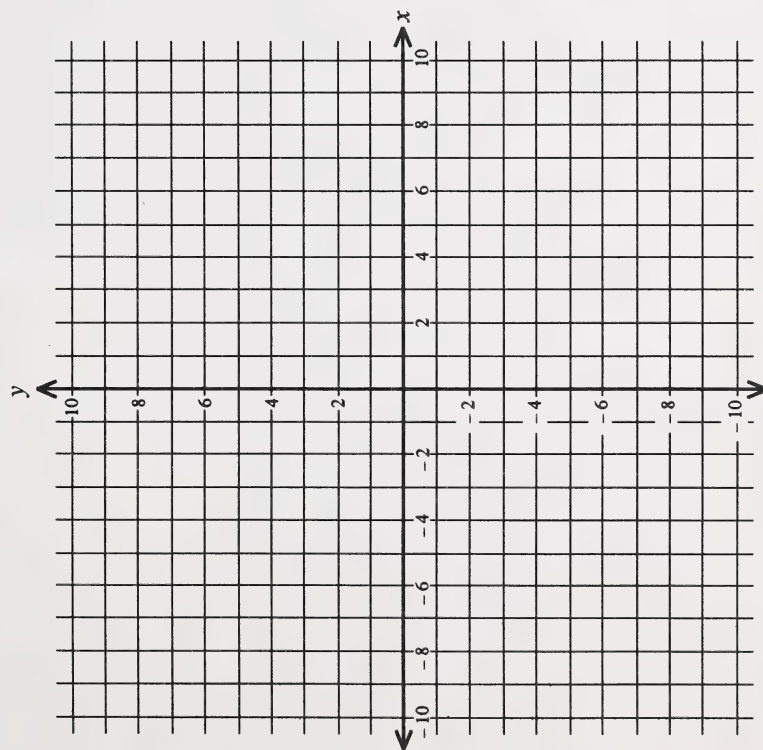


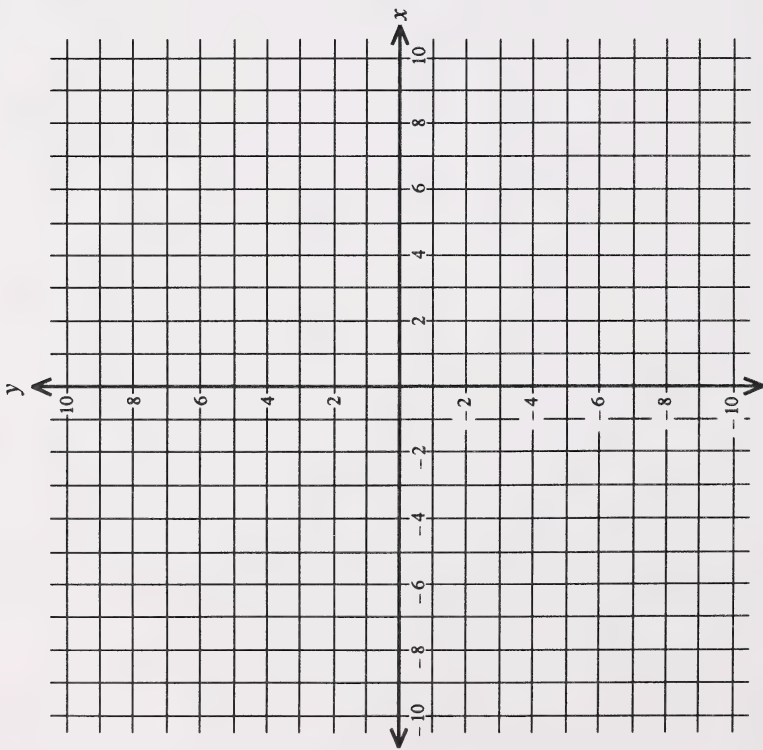
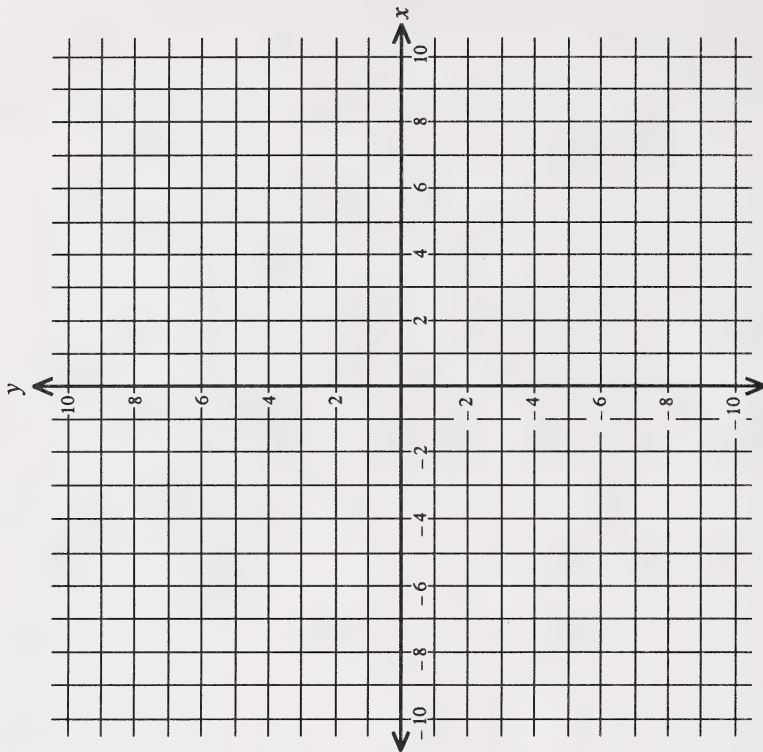


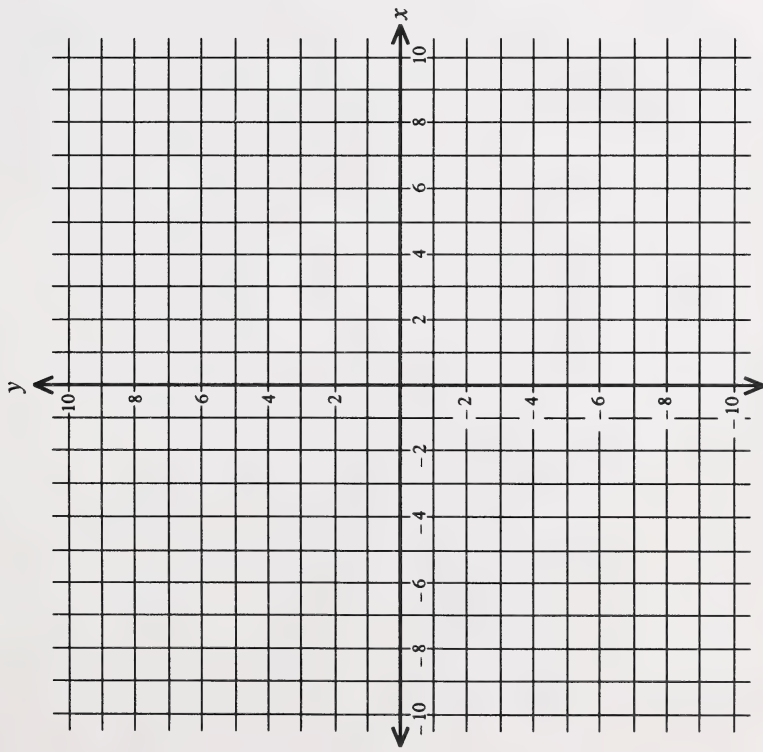
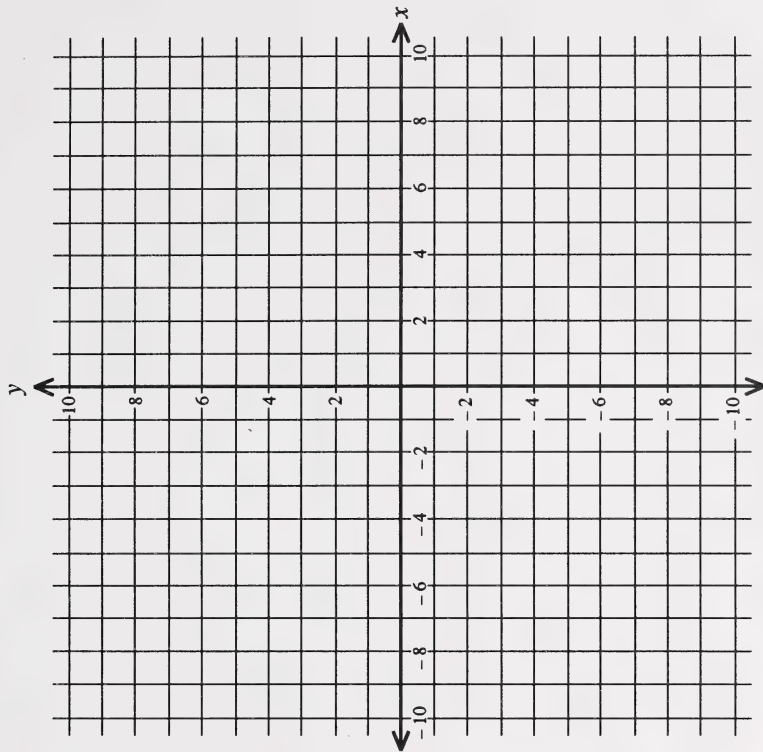


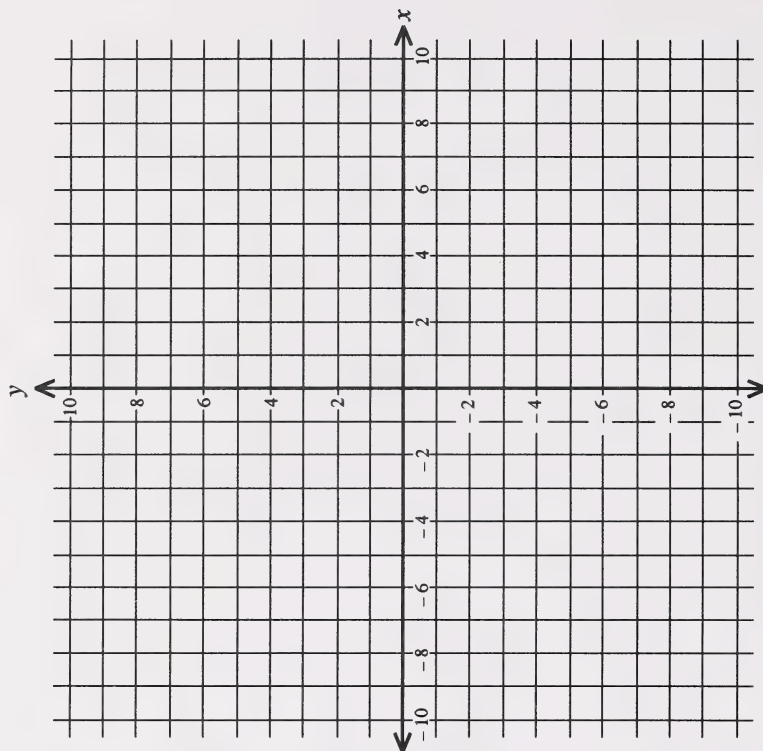
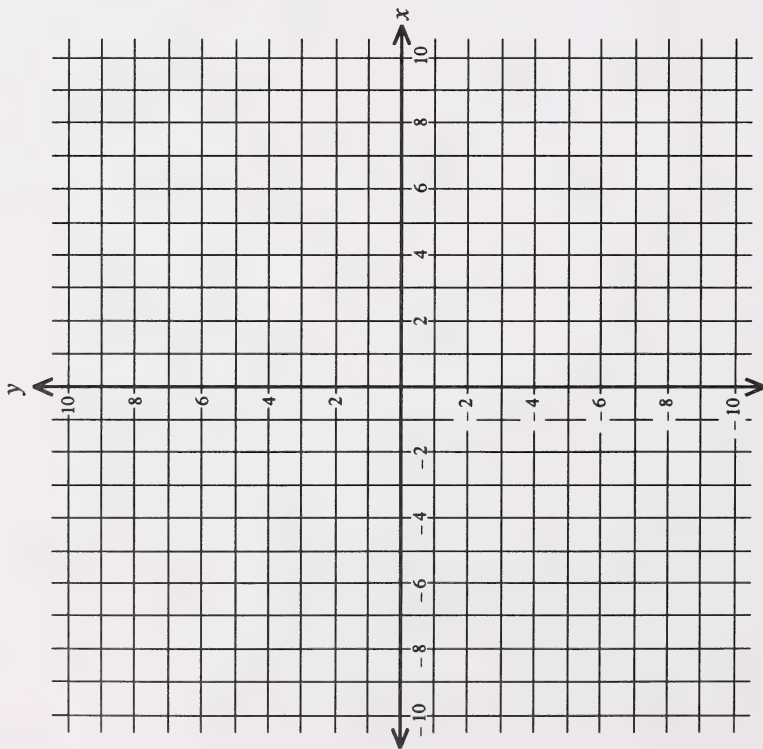


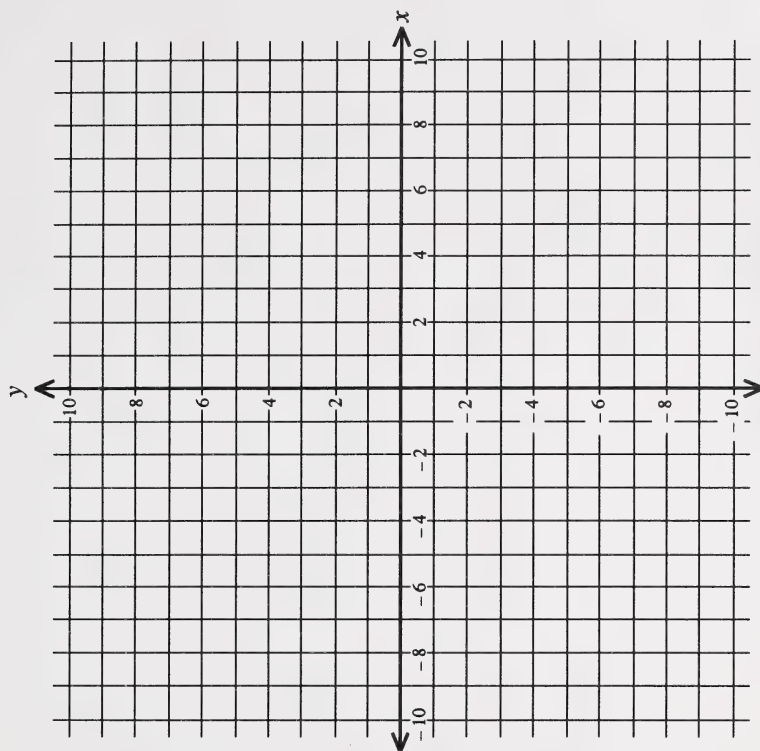
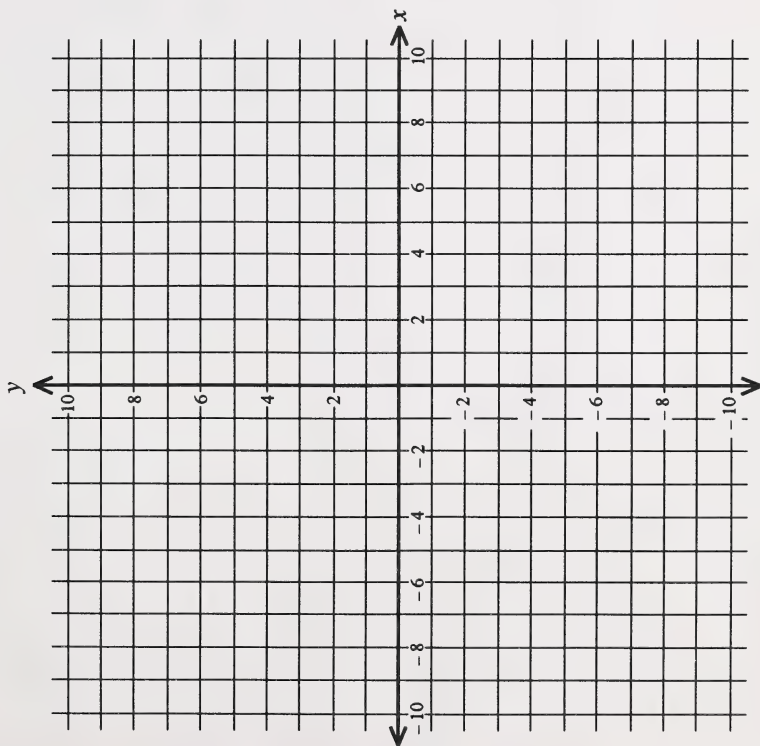


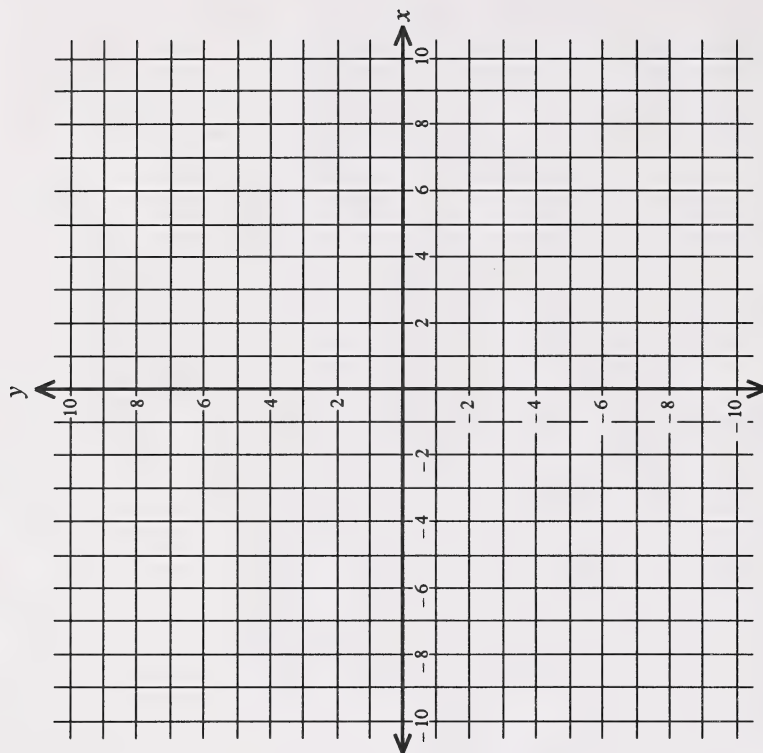
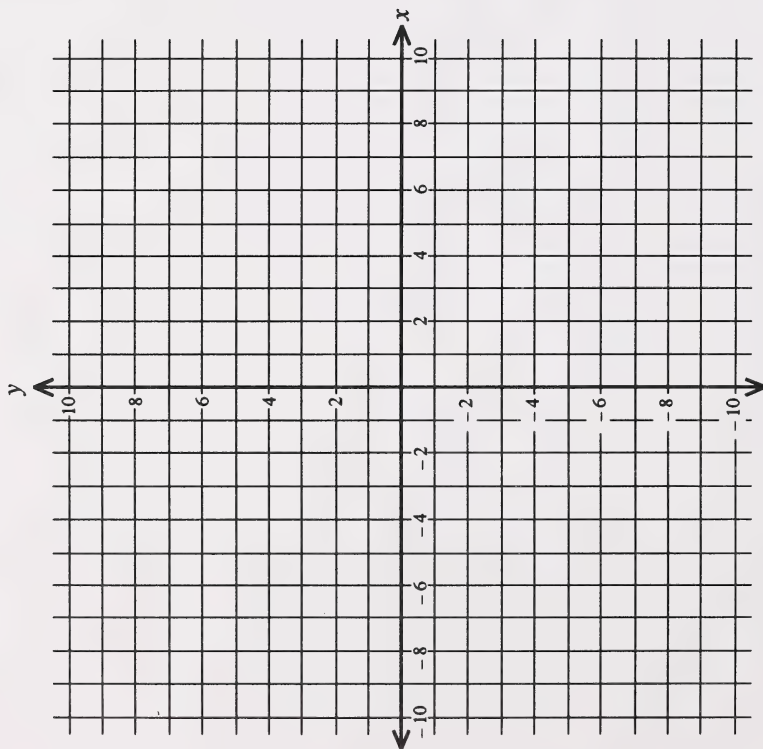


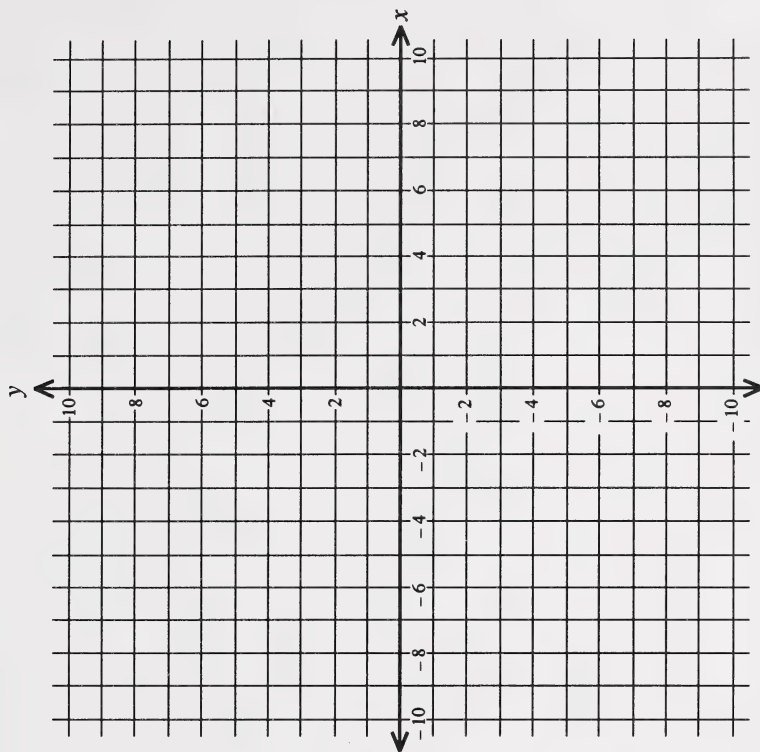
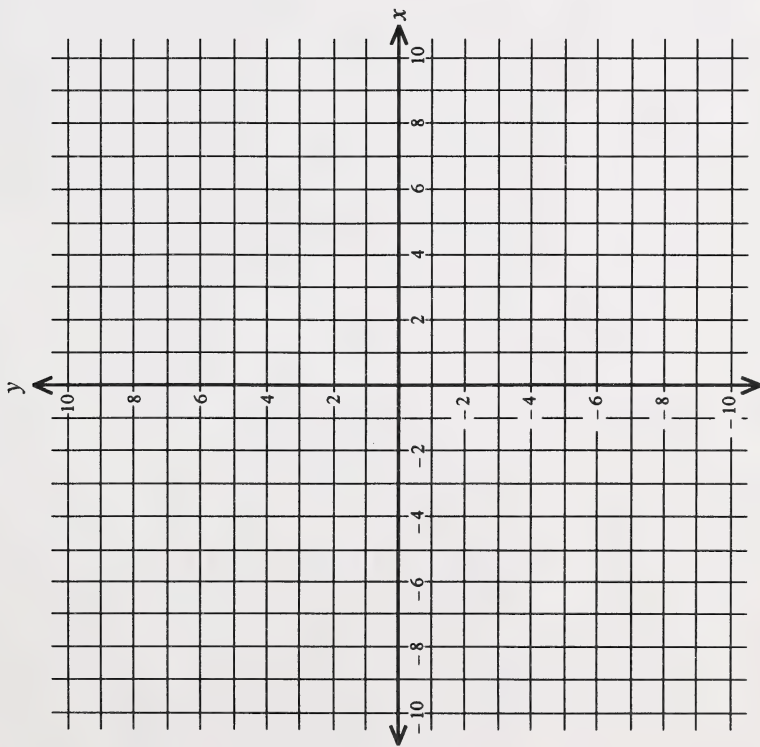


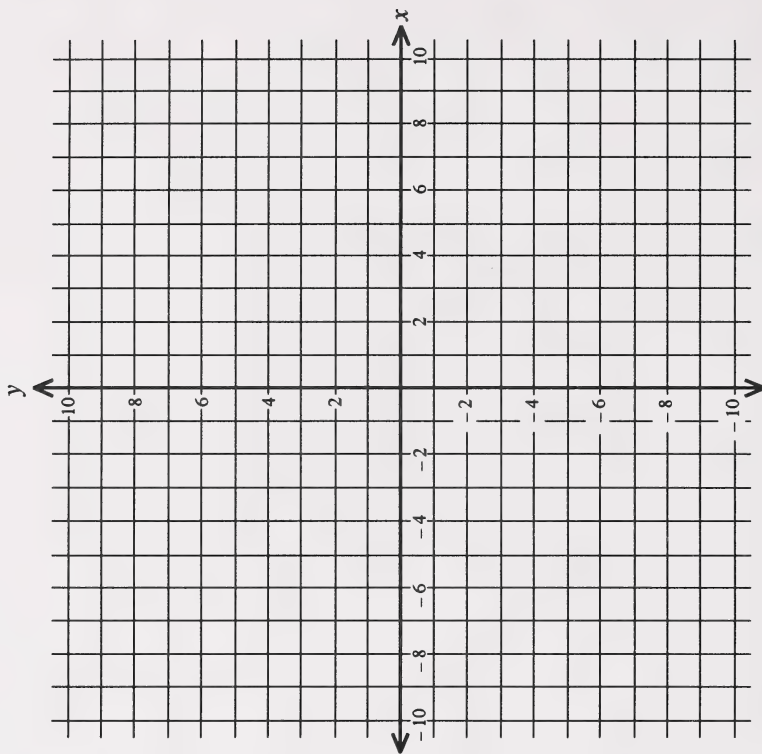
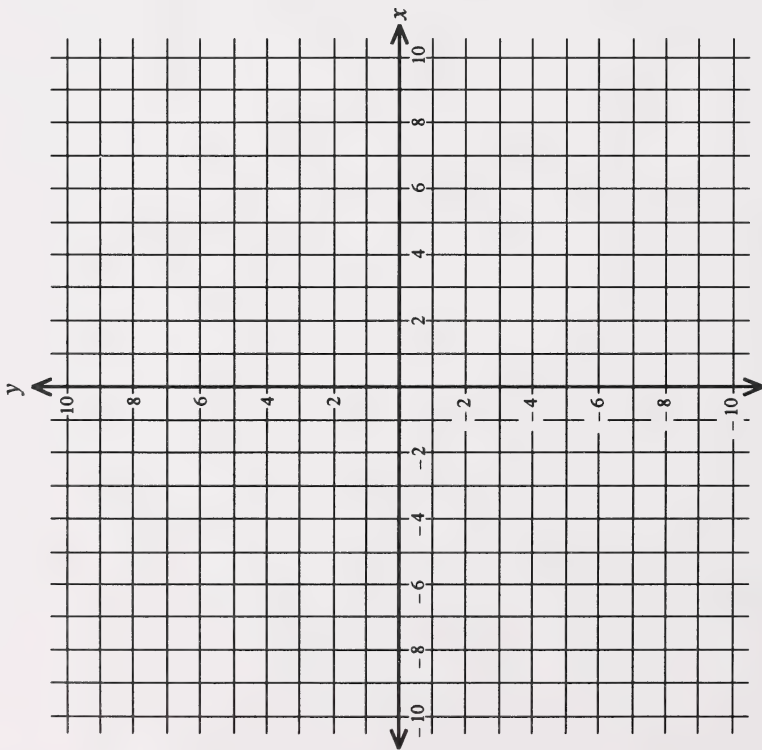


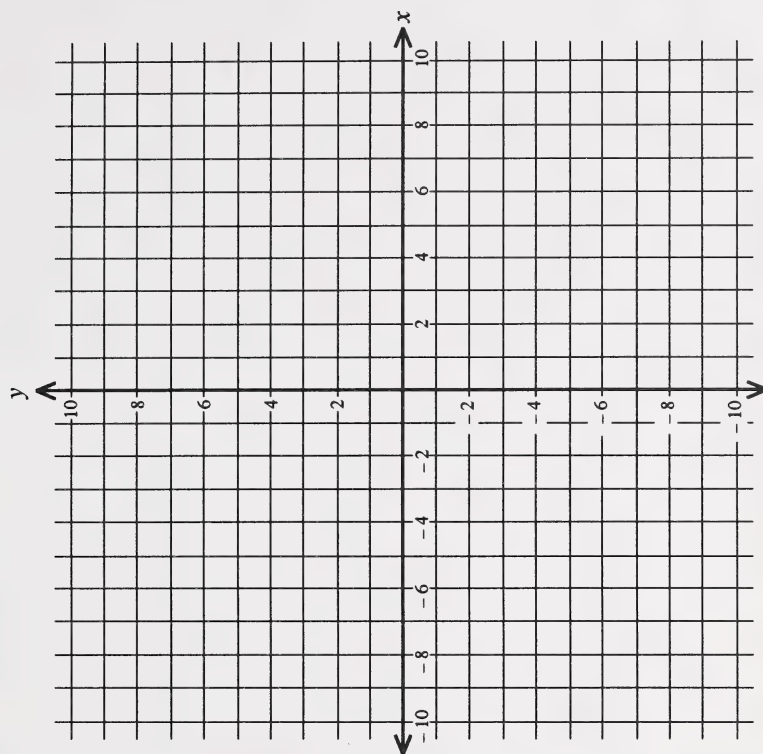
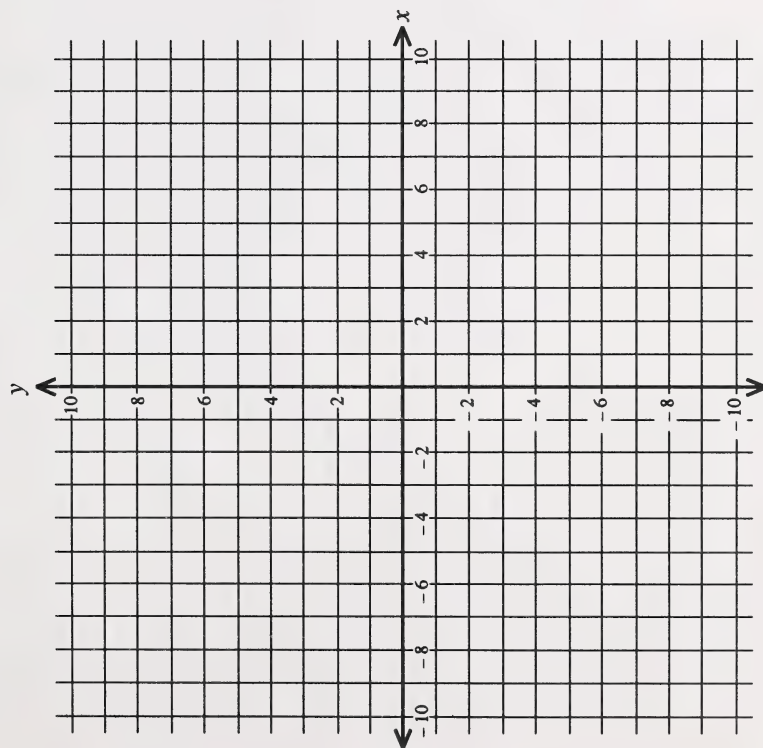


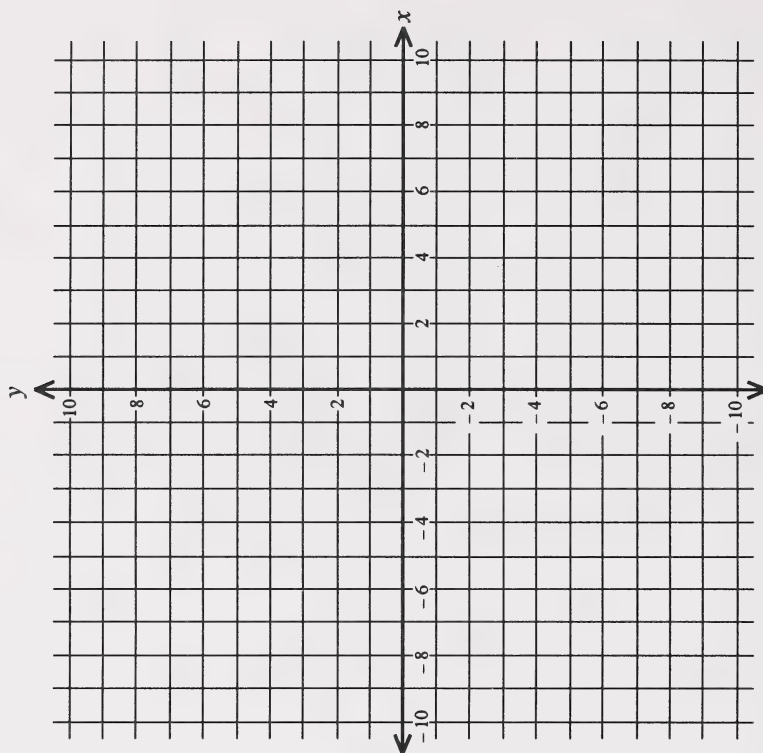
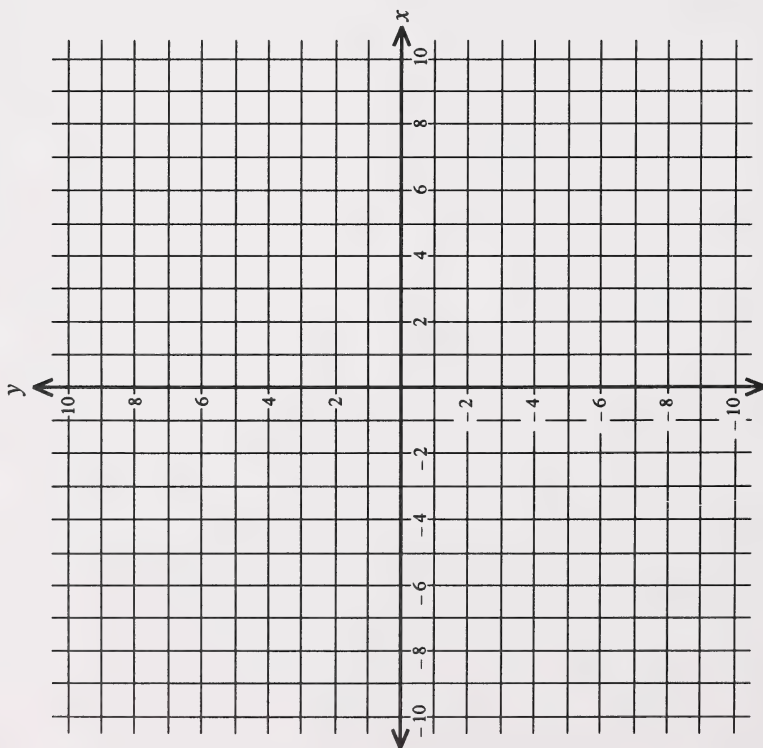


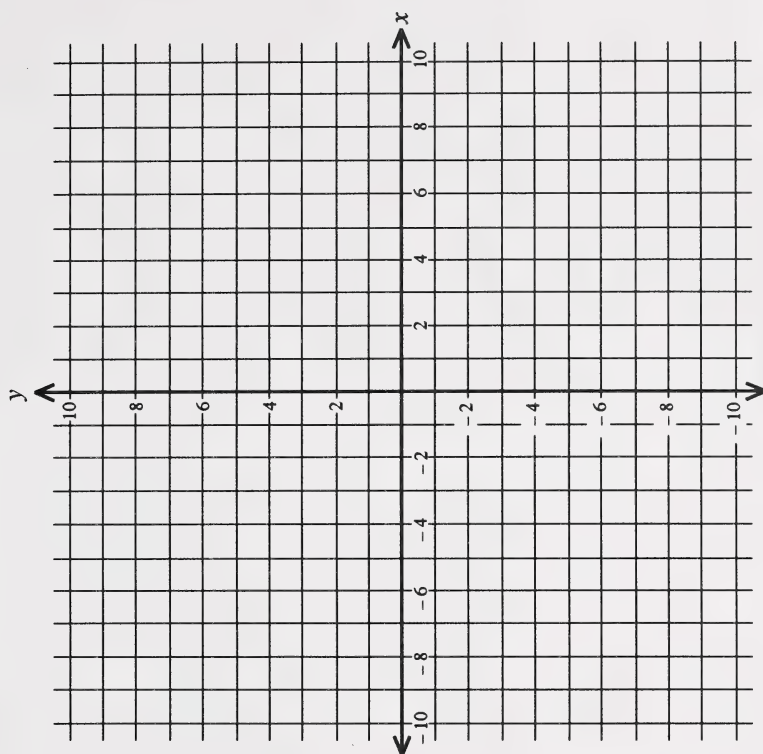
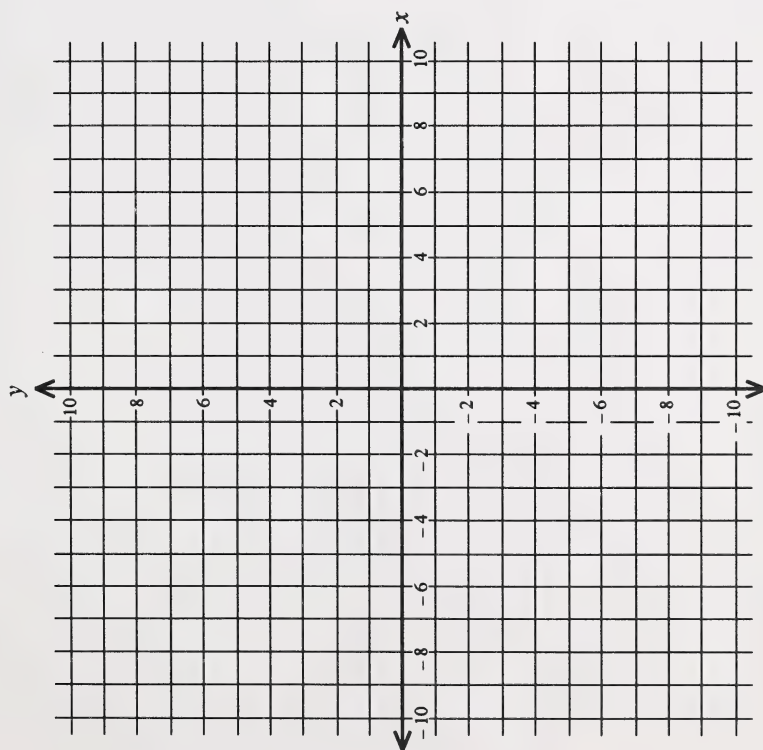


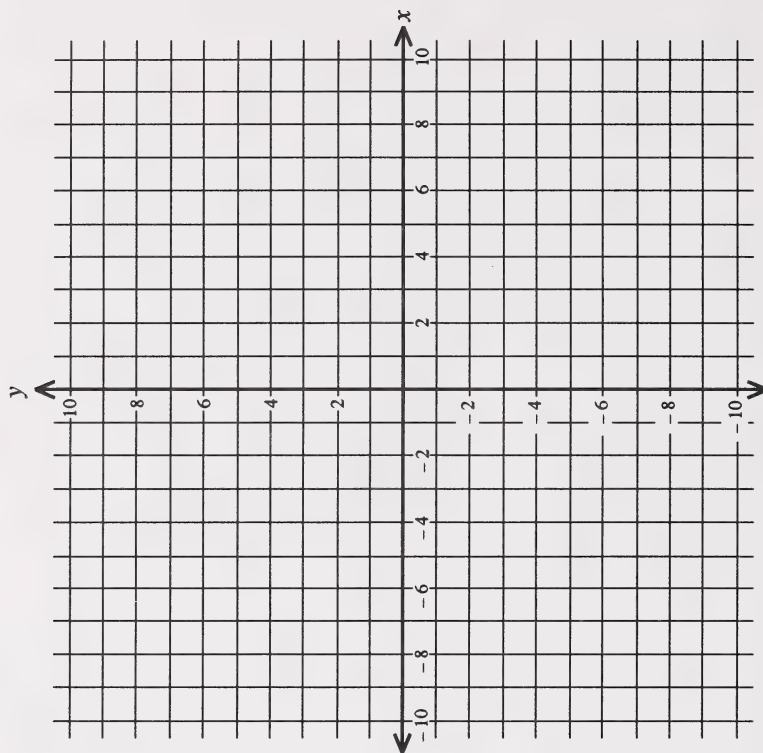
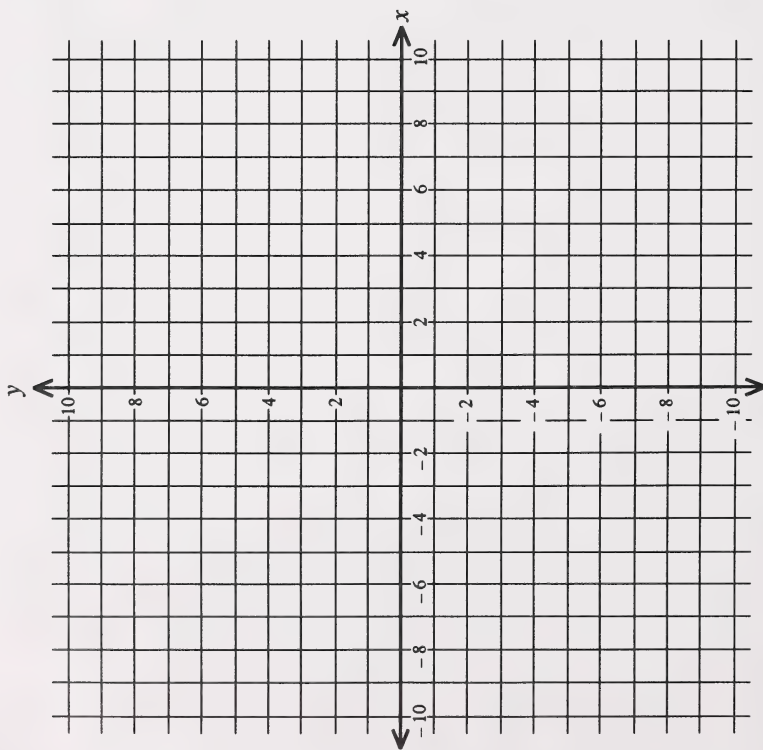


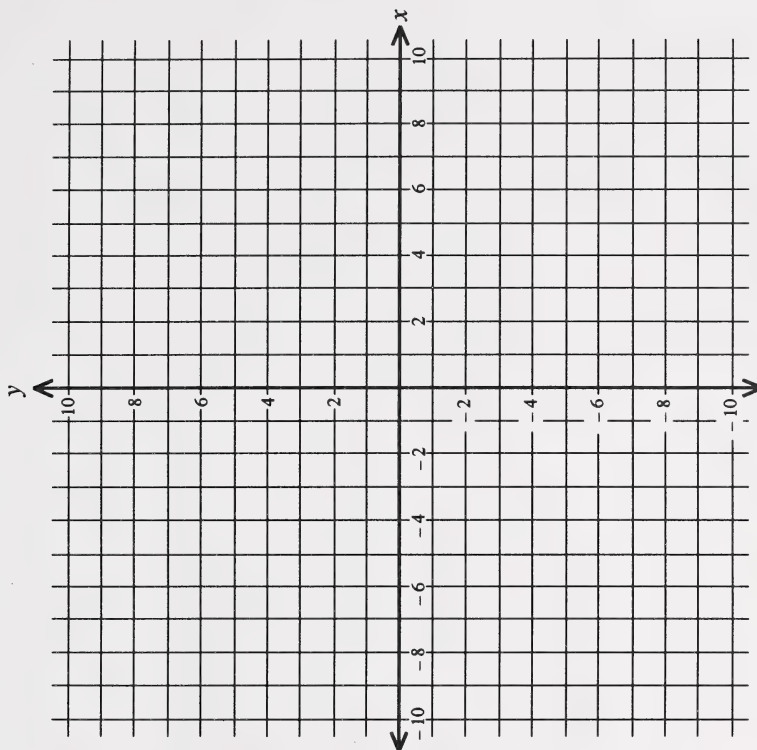
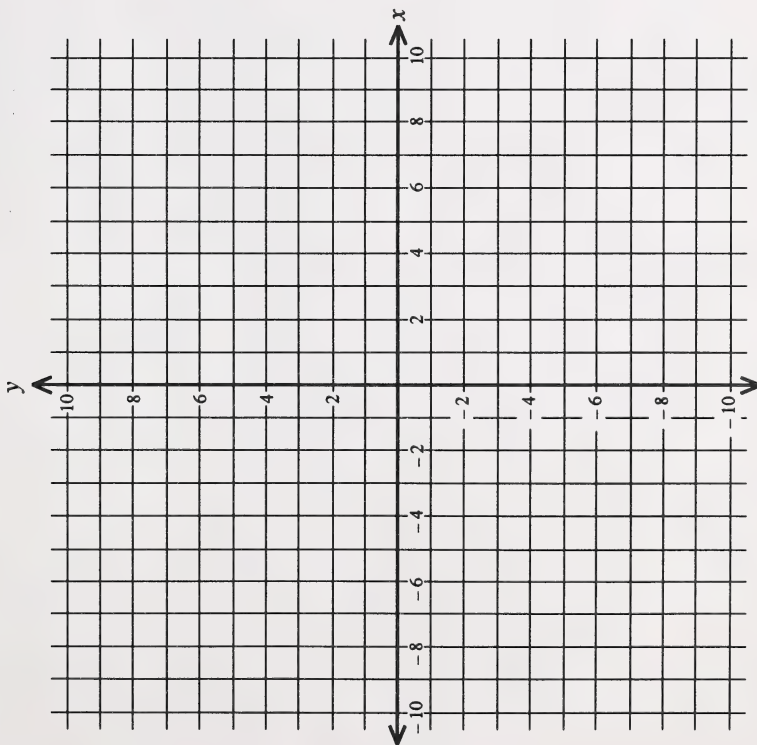


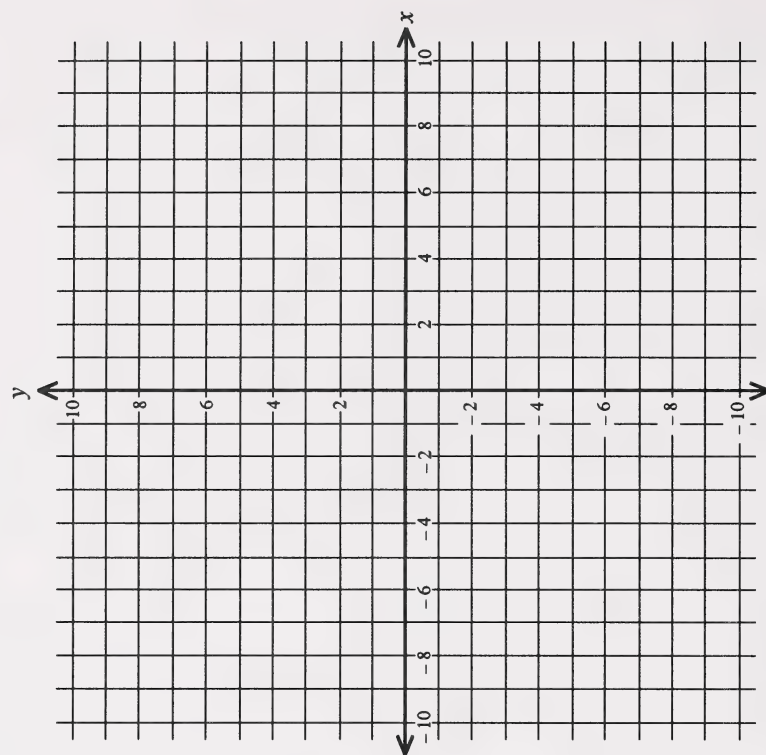
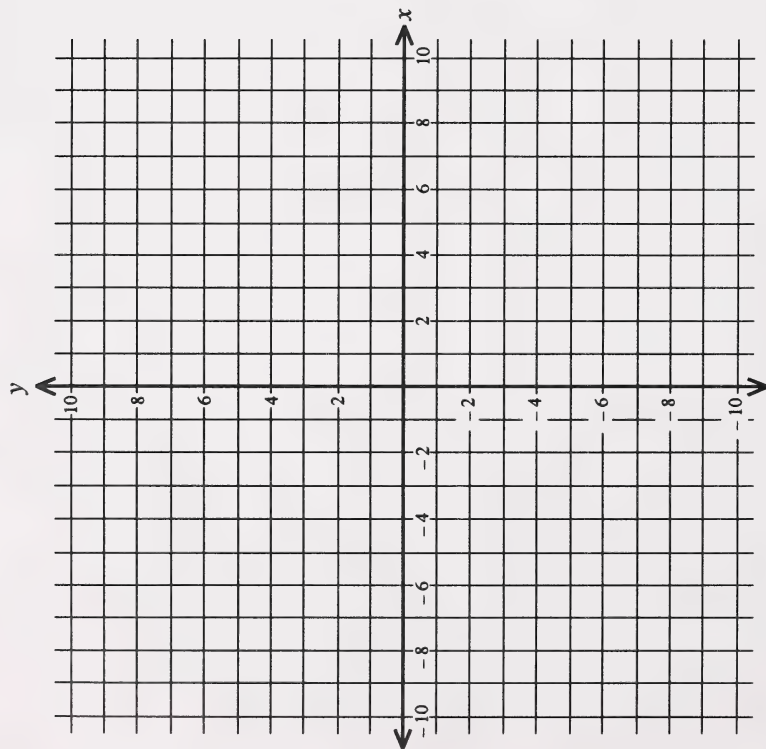


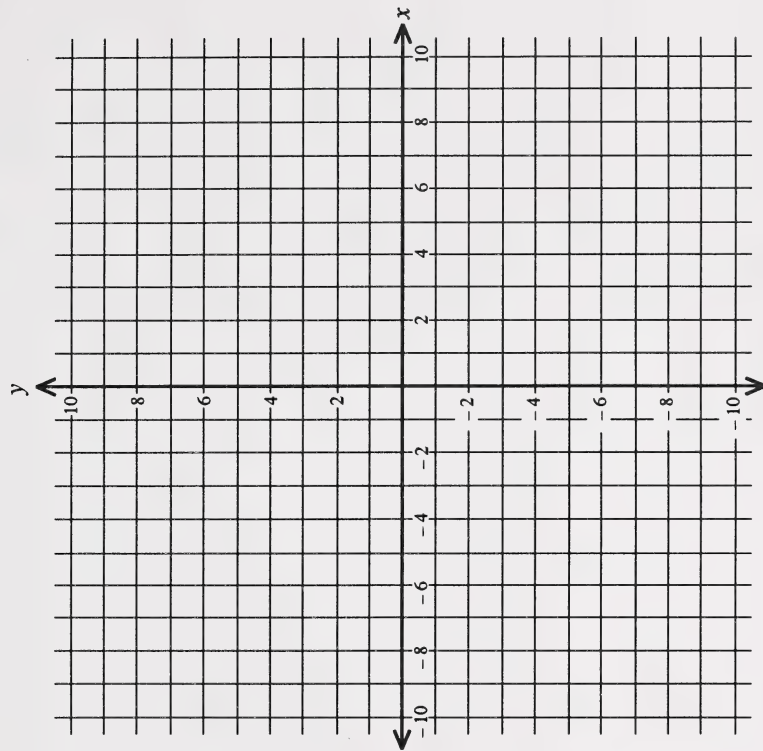
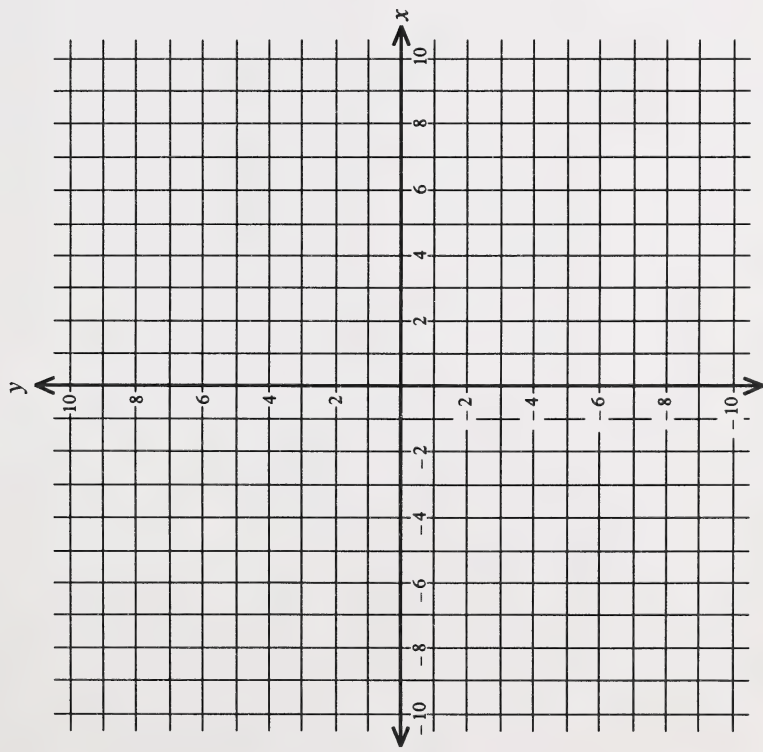


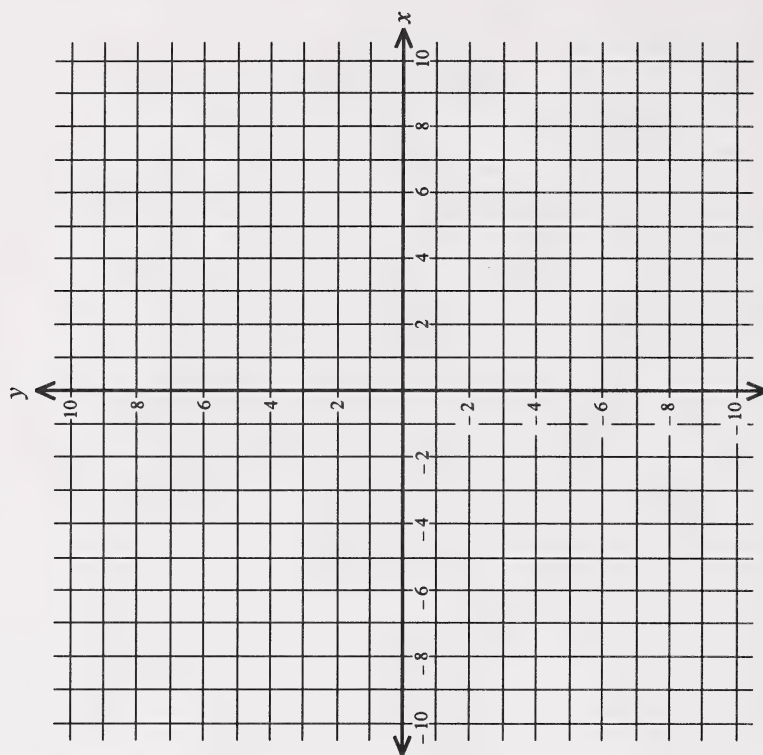
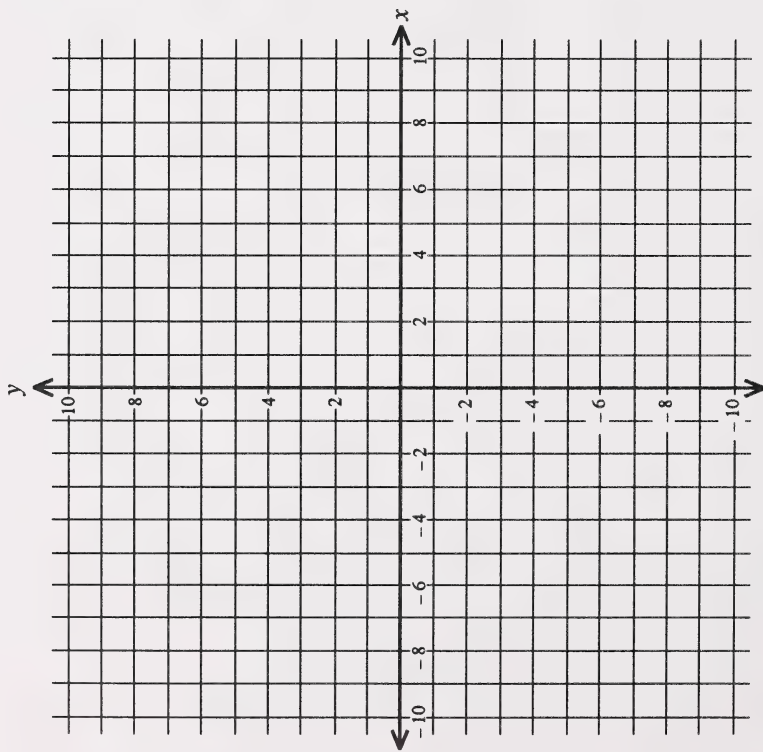


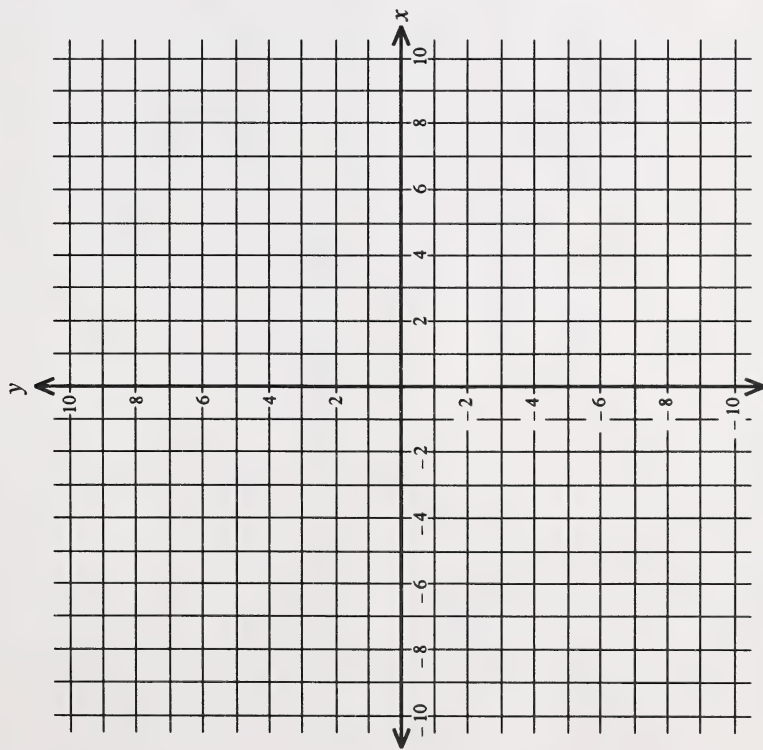
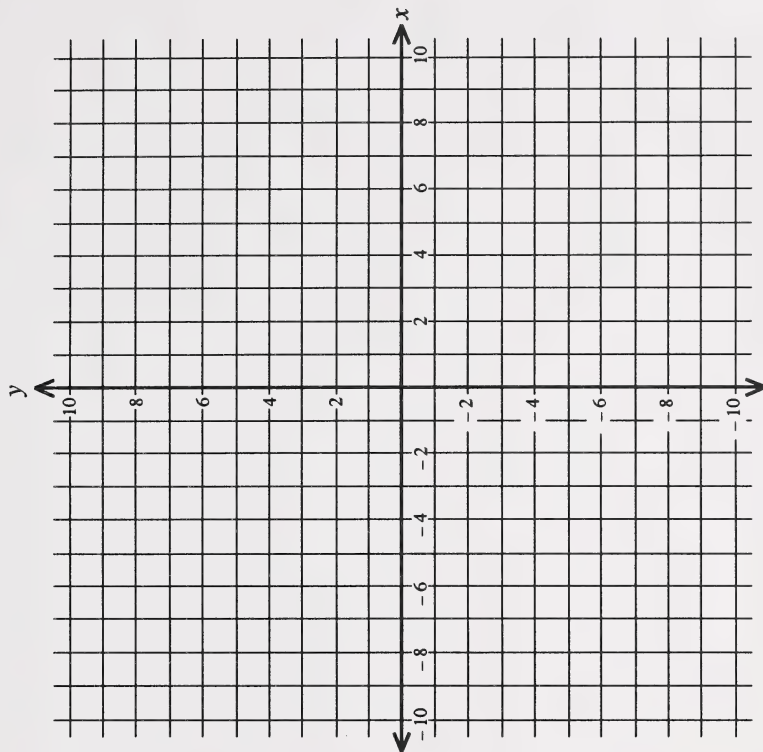


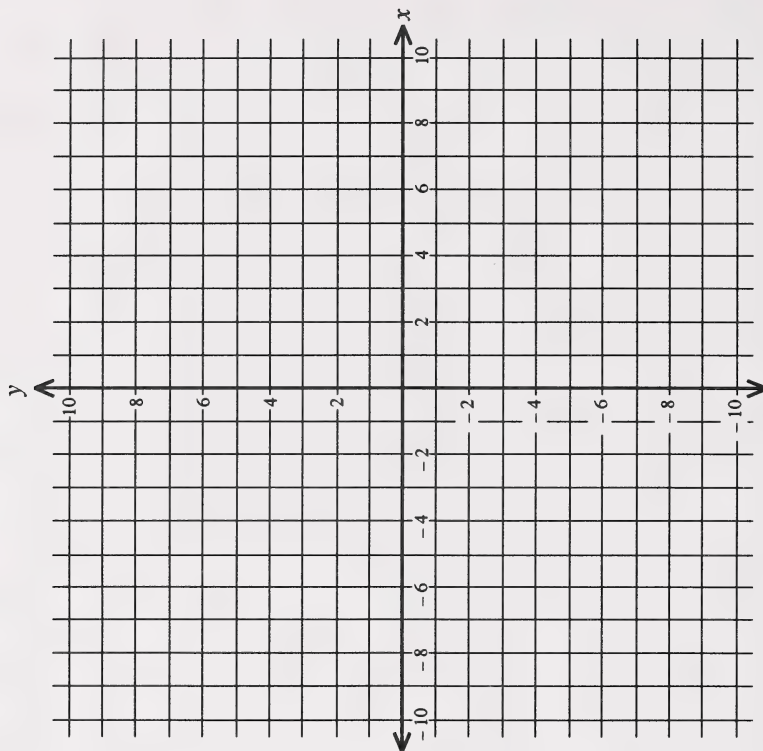
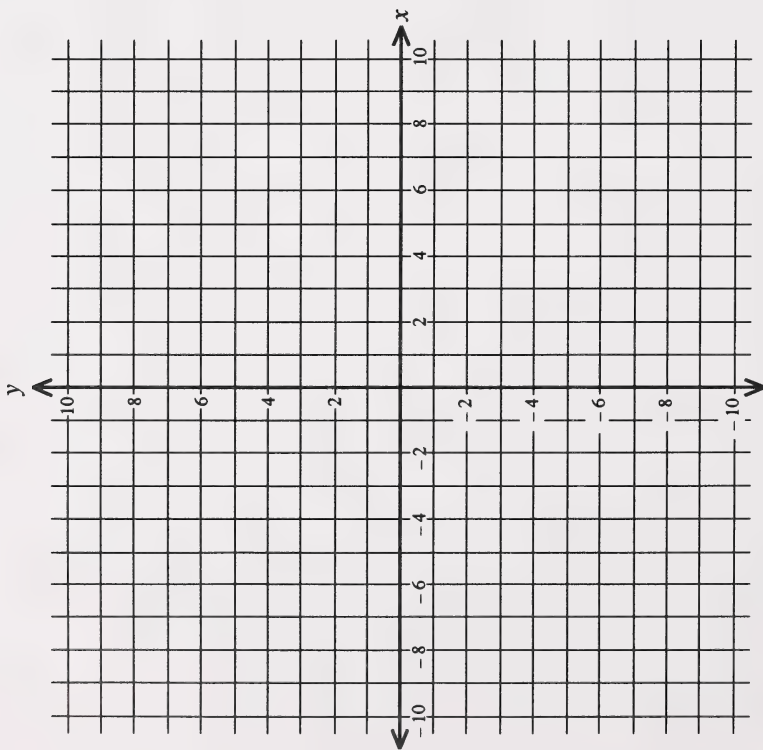


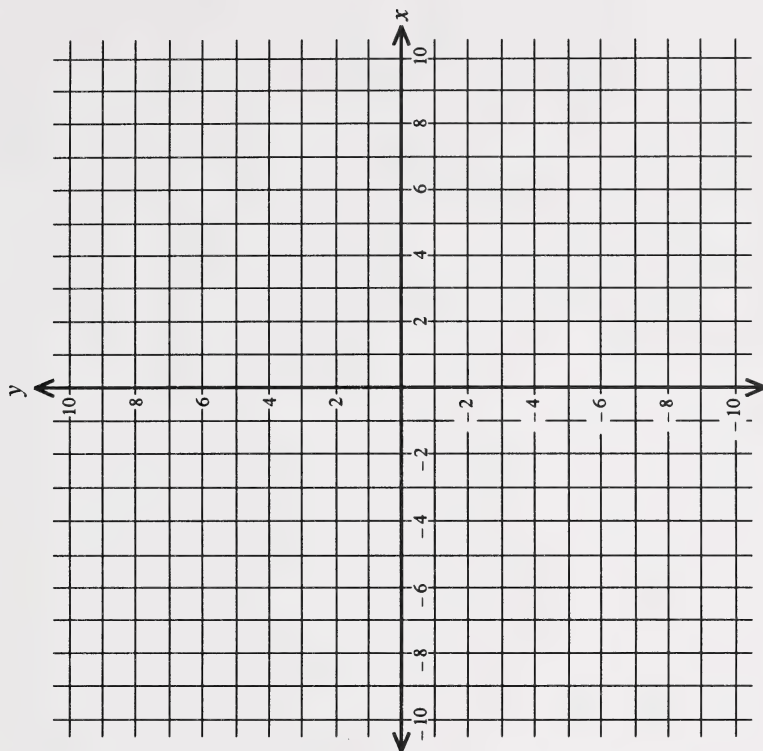
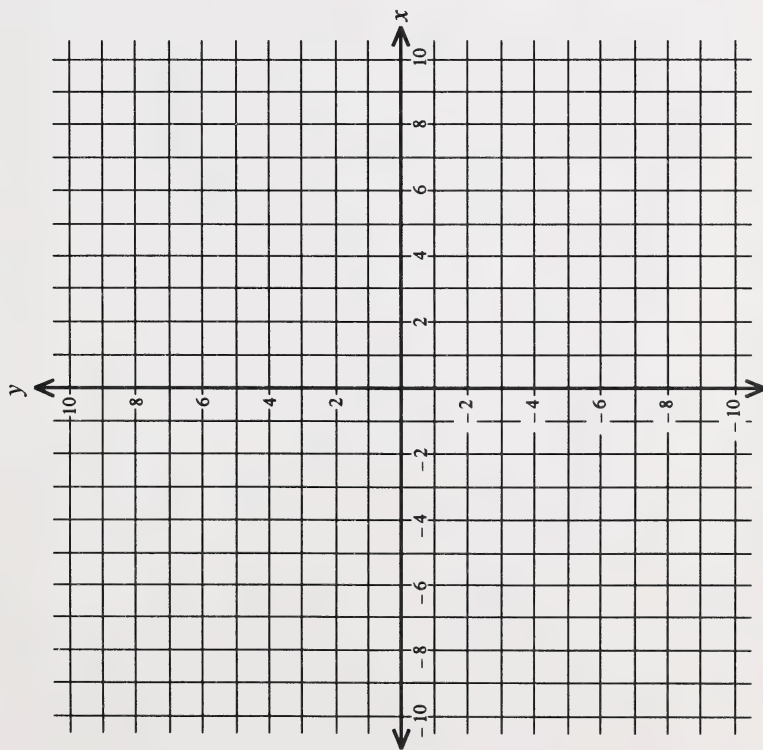


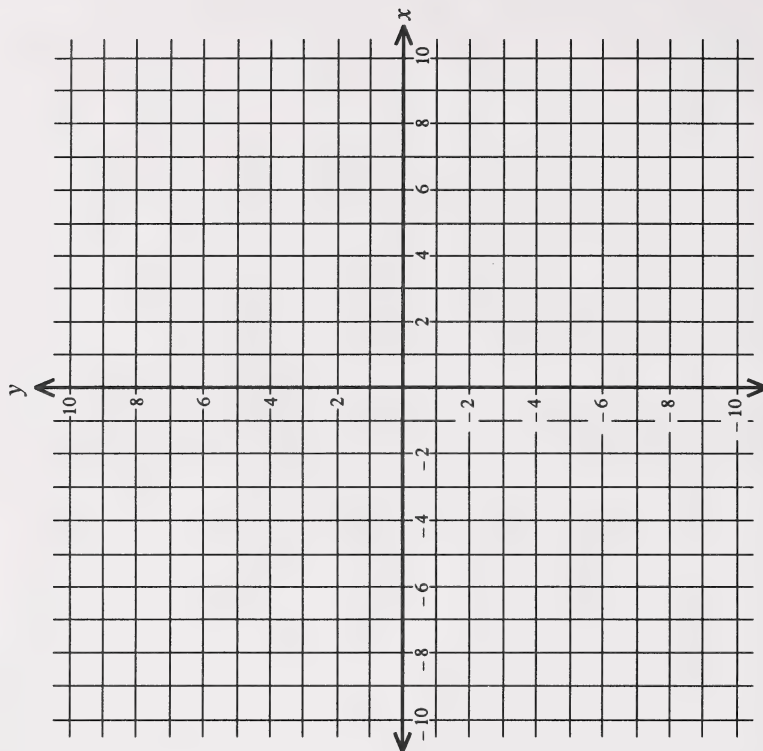
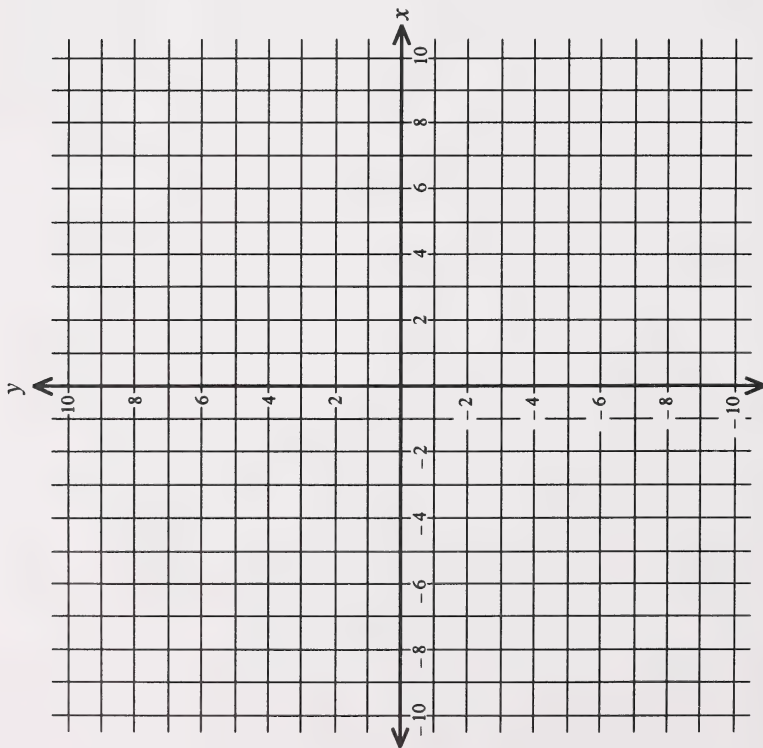


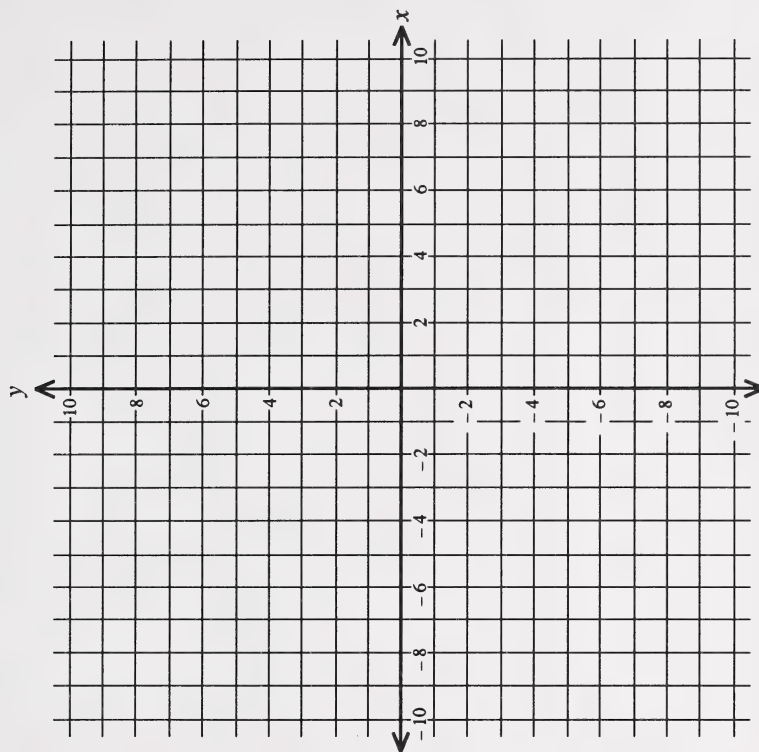
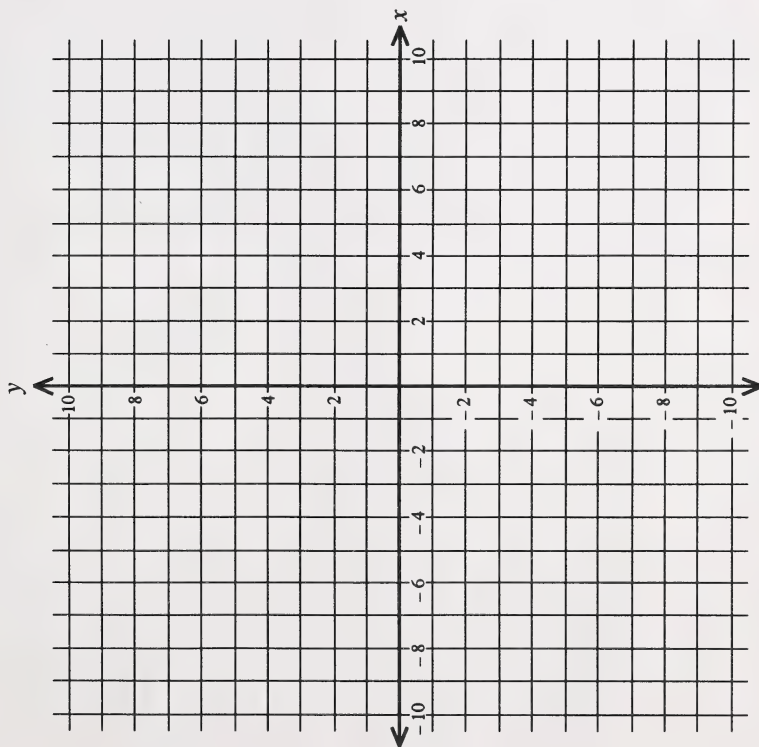


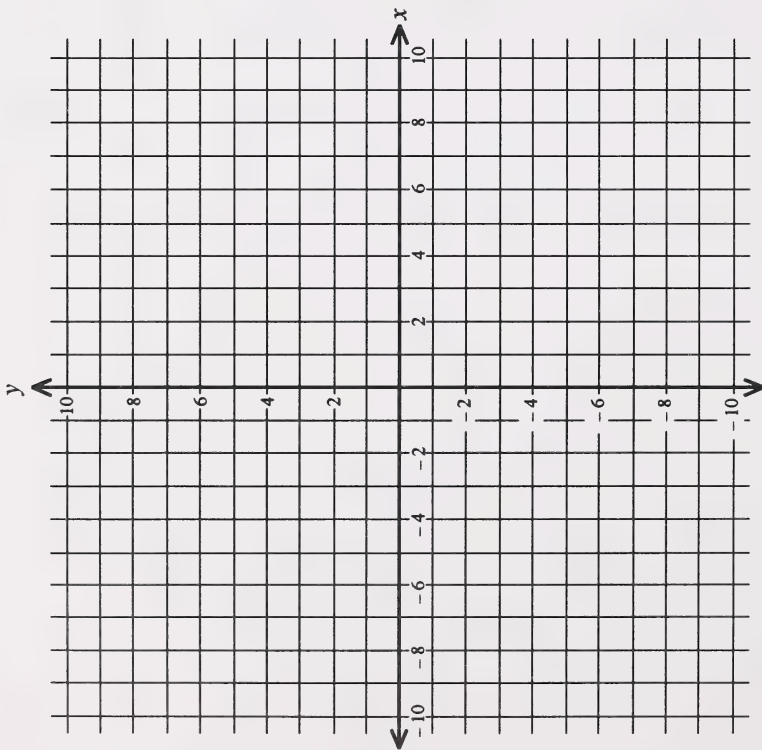
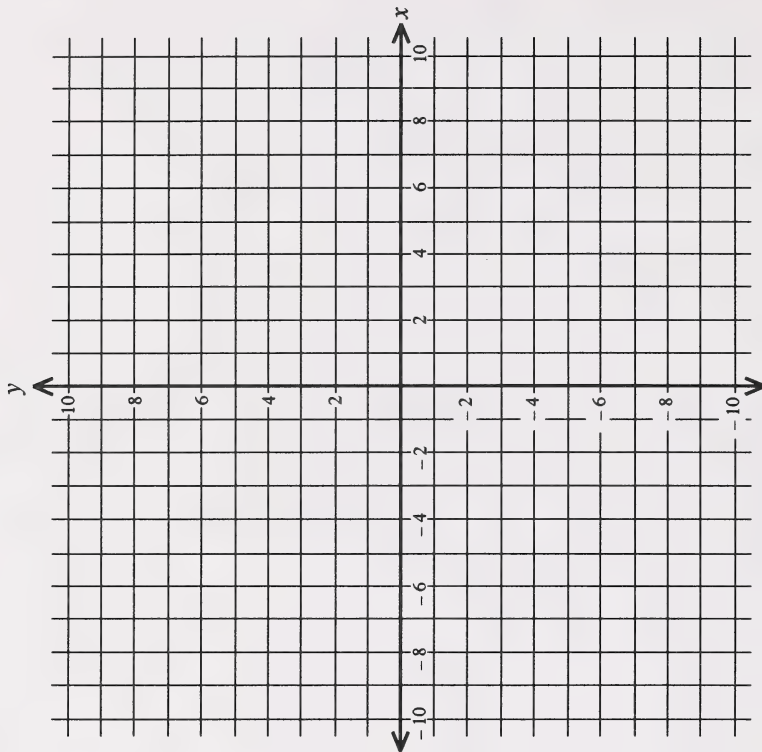












NOTES

NOTES

NOTES

NOTES

NOTES

NOTES



Mathematics 10

Student Module

Unit 5

L.R.D.C.

Producer

1992